Computational Journal of Mathematical and Statistical Sciences
4(2), 424–456
DOI:10.21608/cjmss.2025.375962.1156
https://cjmss.journals.ekb.eg/

Research article

Improved Estimation based on Ranked Set Sampling for the Chris-Jerry Distribution with Application to Engineering Data

Amal S. Hassan[1], **Diaa S. Metwally**[2], **H. E. Semary**[3], **Sid Ahmed Benchiha**[4,*], **Ahmed M. Gemeay**[5], **Mohammed Elgarhy**[6]

[1] Faculty of Graduate Studies for Statistical Research, Cairo University, Giza 12613, Egypt; amal52_soliman@cu.edu.eg.

[2] Department of Accounting, Faculty of Business, Imam Mohammad Ibn Saud Islamic University (IMSIU), Riyadh 11432, Saudi Arabia; dmetwally@imamu.edu.sa.

[3] Department of Mathematics and Statistics, Faculty of Science, Imam Mohammad Ibn Saud Islamic University (IMSIU), Riyadh 11432, Saudi Arabia; hesemary@imamu.edu.sa.

[4] Laboratory of Statistics and Stochastic Processes, University of Djillali Liabes, BP 89, Sidi Bel Abbes 22000, Algeria.

[5] Department of Mathematics, Faculty of Science, Tanta University, Tanta 31527, Egypt; ahmed.gemeay@science.tanta.edu.eg

[6] Department of Basic Sciences, Higher Institute of Administrative Sciences, Belbeis, AlSharkia, Egypt.

* **Correspondence:** benchiha_sidahmed@yahoo.fr

Abstract: A less expensive and time-consuming option to simple random sampling (SRS) is the ranked set (RSS). Frequently, the RSS method produces more accurate estimators than those employed by SRS. The Chris-Jerry distribution (C-JD) is a new heavy-tailed distribution that helps model many data in human endeavors. The flexibility of the C-JD in capturing various hazard rate shapes ensures its applicability to a wide range of real-world data. This article's primary objective is to examine how well fifteen different estimation techniques, such as maximum and minimum spacing distances methods, Kolmogorov method, and variations of the method of the minimum distances, perform when compared to the maximum likelihood method for parameter estimation of the C-JD based on RSS. We conduct comprehensive simulation research and assess the efficacy of various estimates based on RSS and SRS using many criterion measures. To identify the optimal estimating strategy, partial and overall ranks of the mean estimates, mean squared errors, maximum absolute differences, mean absolute relative errors, average absolute biases, average absolute differences and average squared absolute error based on both designs are provided. The results of our study indicate that the maximum likelihood technique consistently outperforms other strategies, as evidenced by the overall rankings. Because RSS is more efficient than SRS, it is a more effective sampling method with lower accuracy measurements. To explain more, an actual data set about the fatigue life of a certain kind of Kevlar epoxy strand subjected to a constant continuous load at a pressure level of 90 % till the strand fails was examined.

Keywords: Chris-Jerry distribution; Anderson-Darling left tail second order; Kolmogorov method; ranked set sampling; minimum spacing square distance method; maximum absolute difference measure.
Mathematics Subject Classification: 60E05, 62E10, 62F10.
Received: 15 April 2025; Revised: 24 April 2025; Accepted: 26 April 2025; Online: 29 May 2025.

1. Introduction

Choosing an appropriate distribution to describe lifetime data is extremely difficult because of the random nature of lifetime data. Given the stochastic nature of lifespan data, finding an appropriate distribution for modeling them is quite difficult. Almost all fields of study, including the social sciences, demography, engineering, physical sciences, finance, insurance, and literature, depend on the analysis and modeling of lifetime data. The Lindley distribution presented by Lindley [1] and the exponential distribution are the two classical one-parameter lifetime distributions used for analyzing the lifetime data. In a thorough comparison of the Lindley and exponential distributions' goodness of fit, Shanker et al. [2] found that while the Lindley distribution provides a much better fit than the exponential distribution in some datasets, the exponential distribution provides a better fit in others, and there were datasets in which neither distribution provided a good fit. The fact that the hazard rate for an exponential distribution is constant, but not for a Lindley distribution, is one of the major benefits of the Lindley distribution over the exponential distribution. Many authors have suggested one-parameter models that follow the Lindley distribution pattern. Some of these models are the Akash distribution [3, 4], the new Lindley distribution [5], the Aradhana distribution [6], the Sujatha distribution [7], the Ishita distribution [8], the xgamma distribution [9, 10], the Rama distribution [11], the Shanker distribution [12], the Rani distribution [13], and the Pranav distribution [14], each of which has a different mixing percentage.

1.1. An Overview of Chris-Jerry Distribution

Recently, the Chris-Jerry distribution (C-JD), which has been assigned by Onyekwere and Obulezi [15], is a one-parameter lifetime distribution that is becoming more and more well-known in statistical literature. The C-JD is intended to model lifetime-representative data, including component lifespans, patient survival periods, and other positive-valued durations. It can capture a variety of shapes and behaviors that may not be sufficiently represented by a single exponential or gamma distribution due to its mixture structure. In domains such as survival analysis and reliability engineering, where lifetime data is essential, this is a major benefit. Compared to multi-parameter distributions, the C-JD relies on a single parameter, which can result in simpler interpretation and more stable parameter estimates. The C-JD have increasing hazard rate functions, and this is an attractive property for modeling processes where risk of failure increases with time. In applications where a simpler model is desired, this simplicity can be especially helpful Onyekwere and Obulezi [15] demonstrated the superiority of the C-JD in fitting data from several sectors of human endeavor when compared to a number of rival distributions, including the Lindley, exponential, Akash, Aradhana, Sujatha, Ishita, xgamma, Rama, Shanker, Rani, and Pranav distributions. The C-JD is derived from a two-component mixture of an exponential distribution with a scale parameter (α) and a gamma distribution with a shape parameter (3) and a scale parameter (α) with the following probability density function (PDF):

$$h(v;\alpha) = \frac{\alpha^2 e^{-\alpha v}(\alpha v^2 + 1)}{\alpha + 2}; \quad v, \alpha > 0. \tag{1.1}$$

The cumulative distribution function (CDF) and hazard rate function (HF) of the C-JD are given, respectively, by

$$H(v;\alpha) = 1 - e^{-\alpha v}\left(\frac{\alpha v(\alpha v+2)}{\alpha+2} + 1\right); \quad v,\alpha > 0, \tag{1.2}$$

and

$$\varsigma(v;\alpha) = \frac{\alpha^2(\alpha v^2+1)}{\alpha^2 v^2 + 2\alpha v + \alpha + 2}.$$

It is clear from the HF that $\varsigma(0) = h(0) = \frac{\alpha^2}{\alpha+2}$, which it is equivalent to the Lindley distribution, and $\varsigma(\infty) = 0$. Plots of the PDF and HF of C-JD are represented in Figures 1 and 2. The left panel of Figure 1 displays reversed-J shapes at $\alpha = 1.5, 2.5$, and 3.5, and decreasing at $\alpha = 0.9$ and 1.2. While the right panel of Figure 1 displays approximately symmetrical at $\alpha = 0.1$, and unimodal forms at $\alpha = 0.15, 0.2$, and 0.25. The HF depicted in Figure 2 can exhibit increasing for $\alpha = 0.1, 0.15, 0.3, 0.25, 0.3$ or approximately U-shaped patterns for $\alpha = 1.2, 1.5, 1.8, 2$, providing a versatile modeling tool compared to other distributions [15]. The C-JD's modeling flexibility is evident by the variety of HF shapes presented in Figure 2. This capability will be demonstrated with a relevant dataset in the application section.

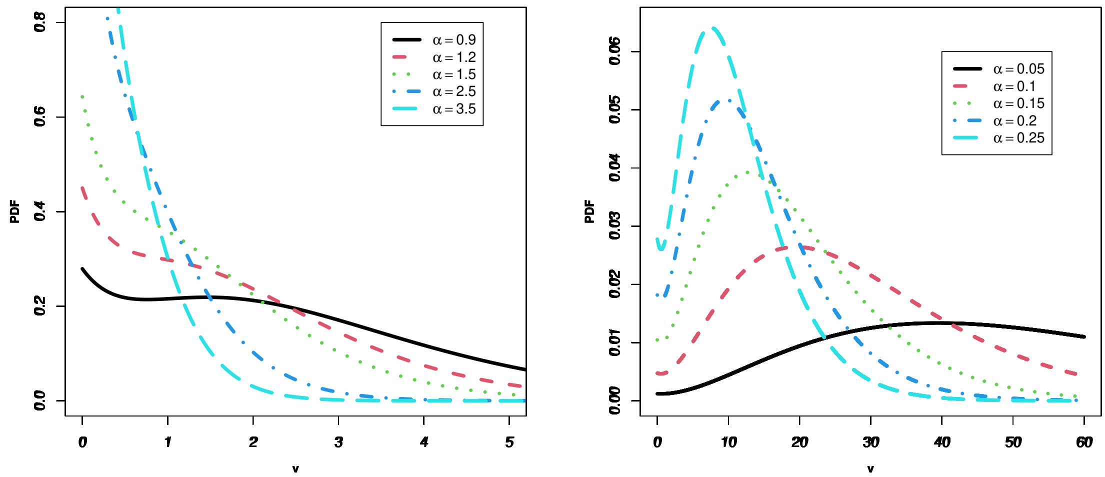

Figure 1. Plots of PDF for the C-JD.

1.2. RSS Description

McIntyre [16] suggested the ranked set sampling (RSS) approach as a low-cost and efficient way to estimate pasture output. Indeed, it's a fascinating advancement in data gathering methods that allows for the acquisition of a more informative sample than would be possible with simple random sampling (SRS). In their work, Takahasi and Wakimoto [17] established the mathematical theory of RSS, proving that the RSS mean estimator, under perfect ranking conditions, is unbiased and has a lower variance than the SRS mean estimator. RSS was shown to be more efficient than SRS, even with ranking errors, by Dell and Clutter [18]. The use of rank-based sampling designs, which are strong substitutes for SRS and frequently result in significant enhancements in precision, has been applied in a variety of contexts.

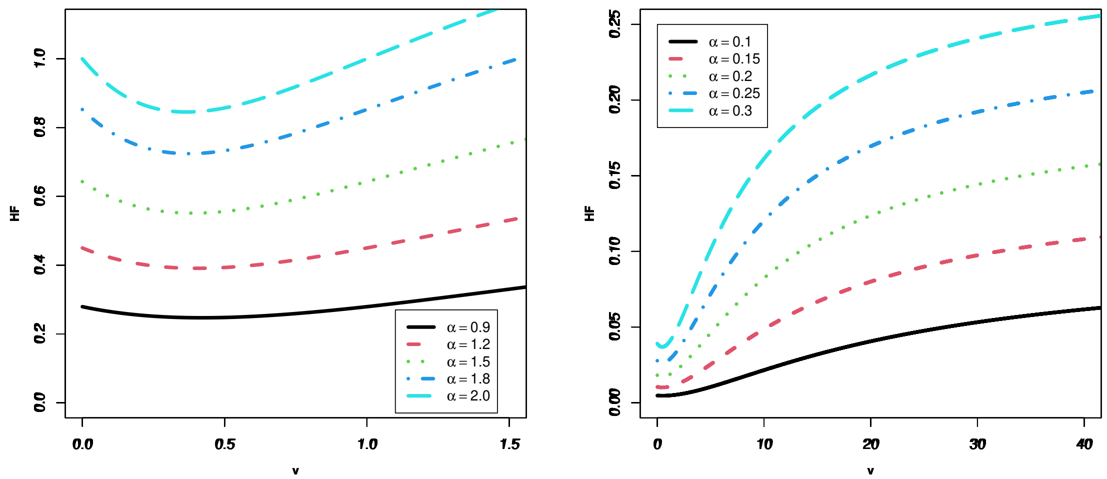

Figure 2. Plots of HF for the C-JD.

These include ecological and environmental studies ([19, 20]), forestry [21], medical studies ([22]), and reliability studies ([23, 24]), among others. Research into RSS within non-parametric frameworks has garnered significant interest from researchers. Amin and Mahdizadeh [25] explored non-parametric estimation of entropy within a RSS framework. A regression-type estimator for the population mean, utilizing supplementary information and considering sensitivity issues, was developed by Shahzad et al. [26] under RSS design. A combined and separate difference and ratio estimation approach for the population mean, using stratified RSS, was proposed by Bhushan et al. [27]. Non-parametric variance estimators for RSS were studied by Begum et al. [28] under different RSS schemes. Improved ratio, product, and exponential estimators of the population mean were introduced by Pandey and Singh [29] in RSS. Pandey et al.[30] proposed robust estimators for estimating the population mean in various RSS scenarios.

Despite not being parametric, the RSS approach has been used by several writers to estimate parameters for a wide range of distributions, and they have shown that the estimates produced by the method are more accurate than those derived from the same sample size and SRS. Many authors investigated the parameter estimation of some distributions using RSS. In subsequent years, RSS was employed to estimate the statistical distribution's involved parameter. Chuiv et al. [31] examined the location parameter estimator of the Cauchy distribution using RSS, whereas the estimator of the exponential distribution parameter was examined by Lam et al. [32]. Based on RSS, Sinha et al. [33] examined the estimation of normal and exponential distribution parameters. The logistic distribution parameters were determined by Abu-Dayyeh et al. [34], whereas the Gumbel distribution parameters under RSS were established by Yousef and Al-Subh [35]. Using modified RSS, Shaibu and Muttlak [36] looked at parameter estimation for gamma, exponential, and normal distributions. Dey et al. [37] compared Bayesian estimation methods for Rayleigh distribution parameter under SRS, RSS, and modified RSS. The novel Weibull-Pareto distribution estimators were studied by Samuh et al. [38]. The log-logistic distribution's estimators were covered by He et al. [39], while Esemen and Grler [40] looked at parameter estimation for the generalized Rayleigh distribution.

In addition to above, Qian et al.[41] studied parameter estimation for the Pareto distribution using modified RSS. Jiang and Gui [42] looked at parameter estimators for the Kumaraswamy distribution, respectively. Al-Omari et al. [43] compared the performance of RSS and SRS for estimating the parameters of the xgamma distribution. The reader may consult for further distributions by referring to [44, 45, 46, 47].

1.3. Research Goals

To the best of our knowledge, no existing research has focused on estimating the parameters of the C-JD using RSS. Given RSS's widespread applications and its potential for improved efficiency compared to SRS for a fixed sample size, we prioritize RSS in our analysis. The C-JD's versatility in modeling skewed data encountered in various fields motivated our interest in this distribution. This study aims to conduct a comprehensive evaluation of different frequentist approaches to estimate C-JD parameter. This work can be summarized as follows:

- Under perfect RSS and SRS designs, we evaluate fifteen frequentist methods for estimating the C-JD parameter. These methods include Anderson-Darling (AD), maximum product spacing (MPS), ordinary least squares (OLS), Kolmogorov, Cramér-von-Mises (CVM), minimum spacing square log distance (MNSSLOD), weighted least squares (WLS), right-tail AD (R-AD), minimum spacing Linex distance (MNSLXD), minimum spacing square distance (MNSSD), maximum likelihood (ML), left tail AD (L-AD), minimum spacing absolute log distance (MNSALOD), minimum spacing absolute distance (MNSAD), and AD left tail second order (AD-LSO) .
- A simulation study is performed to evaluate the effectiveness of the proposed RSS-based estimates in comparison to basic SRS estimates using some accuracy metric. Additionally, we provide selection criteria, including partial and overall rankings, to determine the optimal estimation technique.
- Further analysis using real-world data confirms the superior performance of RSS-based estimators compared to basic SRS estimates, even when using the same number of observed units.

This article is organized as follows. Section 2 provides the ML estimate (MLE) and MPS estimate (MPSE) of the C-JD based on RSS. Different minimum spacing distance estimators of the C-JD are discussed in Section 3. Section 4 provides the five different minimum distance estimators for the C-JD. Section 5 offers a few more estimators. In Section 6, Monte Carlo simulation is used to compare the performance of the RSS-based estimators. Section 7 offers further details based on real data. The final section includes a few final remarks.

2. Maximum Likelihood and Maximum Product Spacing Estimators

This section aims to provide a detailed description of the RSS and derive the MLE and MPSE of the C-JD parameter using the RSS technique.

2.1. Description of RSS Scheme

Designing an RSS design requires choosing a set size, which should not exceed five [48], in order to reduce ranking error. Suppose that the random sample $V = (V_1, V_2, ..., V_q)$, $s = 1, 2, \dots, q$ with the same CDF $H(v)$ and PDF $h(v)$. The RSS technique may be explained as follows:

1. From the target population, q^2 sample units are chosen at random and then distributed into q sets of size q.
$$\begin{matrix} V_{1,1}, & V_{2,1}, & \cdots, & V_{q,1}. \\ \vdots & \vdots & \vdots & \vdots \\ V_{1,q}, & V_{2,q}, & \cdots, & V_{q,q}. \end{matrix}$$
2. Prior to knowing the values for the variable of interest, rank the units within each set. The ranking might be determined by an associated variable connected with an interest variable, or it can be based on expert or personal judgment.
3. In order to pick a sample for real quantification, take the lowest-ranked unit from the first set; the other samples are left unmeasured. Only the second-smallest rank unit from the second row is measured after another sample of size q is picked and arranged. This process is repeated until the element with the greatest rank unit from the qth row is selected. This is described as follows:
$$\begin{matrix} V_{(1),1}, & V_{(2),1}, & \cdots, & V_{(q),1}. \\ \vdots & \vdots & \vdots & \vdots \\ V_{(1),q}, & V_{(2),q}, & \cdots, & V_{(q),q}. \end{matrix}$$
Consequently, one element from each set must be measured for a total of q elements. This procedure is known as a one-cycle RSS of size q. The RSS for this cycle will thus be $V_{(1)1}, V_{(2)2}, ..., V_{(q)q}$. Note that $V_{(s)s}$ stands for the sth order statistic from the sth row, where $(s = 1, 2, \ldots, q)$.
4. To get an RSS of size $n = qd$, repeat steps (1) through (3) d times, or cycles. Note that $V_{(s)s,u}$ $s = 1, \ldots, q$, $u = 1, \ldots, d$ be an RSS.

For simplicity, we write V_{su} instead of $V_{(s)s,u}$ then for fixed u, V_{su}'s are independent with density equal to the same density of the sth order statistic from a sample of size q. The PDF of V_{su} (Arnold et al. [49]), assuming perfect ranking, is as follows, :

$$h(v_{su}) = \frac{1}{B(s, q-s+1)}[H(v_{su})]^{s-1}[1 - H(v_{su})]^{q-s}h(v_{su}), \quad v_{su} \in R, \tag{2.1}$$

where $h(v_{su})$ and $H(v_{su})$ are the PDF and the CDF of V, respectively.

2.2. Maximum Likelihood Estimator

In this subsection, the MLE $\widehat{\alpha}_1$ of α is determined based on RSS methodology. Let $v_{su} = (v_{su}, s = 1, \ldots, q, u = 1, 2, \ldots, d)$ the RSS of size $n = qd$ where q is the set size and d is the number of cycles, gathered from the C-JD. The likelihood function of the C-JD, is obtained by inserting Equations (1.1) and (1.2) in Equation (2.1) as below:

$$L(\alpha) \propto \prod_{s=1}^{q} \prod_{u=1}^{d} [1 - \mathrm{A}(v_{su}, \alpha)]^{s-1}[\mathrm{A}(v_{su}, \alpha)]^{q-s} \frac{\alpha^2 e^{-\alpha v_{su}}(\alpha v_{su}^2 + 1)}{\alpha + 2}, \tag{2.2}$$

where $\mathrm{A}(v_{su}, \alpha) = e^{-\alpha v_{su}} \left(\frac{\alpha v_{su}(\alpha v_{su}+2)}{\alpha+2} + 1 \right)$.

The logarithmic of Equation (2.2), indicated by $l^{\bullet}(\alpha)$, is expressed by:

$$\begin{aligned} l^{\bullet}(\alpha) &\propto 2n\log\alpha - n\log(\alpha+2) \\ &+ \sum_{s=1}^{q}\sum_{u=1}^{d}\left\{(s-1)\log\left[1-\mathrm{A}(v_{su},\alpha)\right] + (q-s)\log\left[\mathrm{A}(v_{su},\alpha)\right] - \alpha v_{su} + \log(\alpha v_{su}^2+1)\right\}. \end{aligned} \tag{2.3}$$

Differentiate Equation (2.3) with respect to α and equate to zero, gives

$$\frac{\partial l^{\bullet}(\alpha)}{\partial\alpha} = \frac{2n}{\alpha} - \frac{n}{\alpha+2} + \sum_{s=1}^{q}\sum_{u=1}^{d}\left[\frac{(s-1)\mathrm{A}'_{\alpha}(v_{su},\alpha)}{[1-\mathrm{A}(v_{su},\alpha)]} + \frac{(q-s)\mathrm{A}'_{\alpha}(v_{su},\alpha)}{\mathrm{A}(v_{su},\alpha)} - v_{su} + \frac{v_{su}^2}{\alpha v_{su}^2+1}\right] = 0, \tag{2.4}$$

where $\mathrm{A}'_{\alpha}(v_{su},\alpha) = \frac{\partial \mathrm{A}_{\alpha}(v_{su},\alpha)}{\partial\alpha} = \frac{-\alpha v_{su}e^{-\alpha v_{su}}}{(\alpha+2)^2}\left[(\alpha+2)\left(\alpha v_{su}^2+1\right)+\alpha v_{su}+2\right]$.

The MLE $\widehat{\alpha}_1$ of α, is the solution of the non-linear equation (2.4). It seems that Equation (2.4) has no analytical solution, therefore, one can use a numerical method for obtaining $\widehat{\alpha}_1$.

2.3. *Maximum Product of Spacings Estimator*

In place of ML method, the MPS approach was presented by Cheng and Amin [50, 51] as an alternative way of estimating the continuous univariate distribution's unknown parameters. In support of our decision, Cheng and Amin [51] demonstrated that this approach is consistent across broader circumstances and equally efficient as the MLE.

To obtain the MPSE $\widehat{\alpha}_2$ of α based on RSS, assuming that $V_{(1:n)}, V_{(2:n)}, ..., V_{(n:n)}$ be an ordered sample forming RSS of size $n = qd$, where q is the set size and d is the number of cycles gathered from the C-JD. The uniform spacings are as follows:

$$\Psi_s(\alpha) = H\left(v_{(s:n)}\,|\alpha\right) - H\left(v_{(s-1:n)}\,|\alpha\right), \quad i = 1,2,...,n+1.$$

Note that$H\left(v_{(s:n)}\,|\alpha\right)$ is the CDF of C-JD with ordered sample, $H\left(v_{(0:n)}\,|\alpha\right) = 0, \quad H\left(v_{(n+1:n)}\,|\alpha\right) = 1$, such that $\sum_{s=1}^{n+1}\Psi_s(\alpha) = 1$.

The MPSE $\widehat{\alpha}_2$ of α is produced by maximizing the following function with respect to α

$$\Im = \frac{1}{n+1}\sum_{s=1}^{n+1}\log\left[\Psi_s(\alpha)\right]. \tag{2.5}$$

To generate the MPSE $\widehat{\alpha}_2$ of α an alternative solution for Equation (2.5) is numerically computed as follows:

$$\frac{\partial\Im}{\partial\alpha} = \frac{1}{n+1}\sum_{s=1}^{n+1}\frac{1}{\Psi_s(\alpha)}\left[\pi\left(v_{(s:n)}\,|\alpha\right) - \pi\left(v_{(s-1:n)}\,|\alpha\right)\right] = 0,$$

where

$$\pi\left(v_{(s:n)}\,|\alpha\right) = \frac{\partial}{\partial\alpha}H\left(v_{(s:n)}\,|\alpha\right) = \frac{e^{-\alpha v_{(s:n)}}}{(\alpha+2)^2}\left[\alpha^2 v_{(s:n)}^2 + \alpha^2 v_{(s:n)}^3 + \alpha^2 v_{(s:n)} + 2\alpha^2 v_{(s:n)}^3 + 2\alpha v_{(s:n)}\right], \tag{2.6}$$

and,

$$\pi\left(v_{(s-1:n)}\,|\alpha\right) = \frac{\partial}{\partial\alpha}H\left(v_{(s-1:n)}\,|\alpha\right) = \frac{e^{-\alpha v_{(s-1:n)}}}{(\alpha+2)^2}\begin{bmatrix}\alpha^2 v_{(s-1:n)}^2 + \alpha^2 v_{(s-1:n)}^3 + \alpha^2 v_{(s-1:n)} \\ +2\alpha^2 v_{(s-1:n)}^3 + 2\alpha v_{(s-1:n)}\end{bmatrix}. \tag{2.7}$$

3. Minimum Spacing Distance Estimators

This section provides the various C-JD parameter estimators including, MNSSD estimate (MNSSDE), MNSALOD estimate (MNSALODE), MNSSLOD estimate (MNSSLODE), MNSLXD estimate (MNSLXDE), and MNSAD estimate (MNSADE) using RSS.
Let $V_{(1:n)}, V_{(2:n)}, ..., V_{(n:n)}$ to be an ordered sample forming RSS of size $n = qd$ collected from the C-JD. The MNSADE $\widehat{\alpha}_3$ of α, MNSALODE $\widehat{\alpha}_4$ of α, MNSSDE $\widehat{\alpha}_5$ of α, MNSSLODE $\widehat{\alpha}_6$ of α, and MNSLXDE $\widehat{\alpha}_7$ of α, are yielded, respectively, by minimizing the following functions:

$$\left.\begin{aligned}
\eta(\alpha) &= \sum_{s=1}^{n+1}\left|\Psi_s(\alpha) - \frac{1}{n+1}\right|, \\
\eta_1(\alpha) &= \sum_{s=1}^{n+1}\left|\log\left(\Psi_s(\alpha)\right) - \log\left(\frac{1}{n+1}\right)\right|, \\
\eta_2(\alpha) &= \sum_{s=1}^{n+1}\left[\Psi_s(\alpha) - \frac{1}{n+1}\right]^2, \\
\eta_3(\alpha) &= \sum_{s=1}^{n+1}\left[\log\Psi_s(\alpha) - \log\left(\frac{1}{n+1}\right)\right]^2, \\
\eta_4(\alpha) &= \sum_{s=1}^{n+1}\left[e^{\Psi_s(\alpha))-\frac{1}{n+1}} - \left(\Psi_s(\alpha) - \frac{1}{n+1}\right) - 1\right]^2
\end{aligned}\right\}. \tag{3.1}$$

As opposed to using Equation (3.1), MNSADE $\widehat{\alpha}_3$, MNSALODE $\widehat{\alpha}_4$, MNSSDE $\widehat{\alpha}_5$, MNSSLODE $\widehat{\alpha}_6$, and MNSLXDE $\widehat{\alpha}_7$, can be obtained by empirically solving the following nonlinear equations.

$$\frac{\partial\eta(\alpha)}{\partial\alpha} = \sum_{s=1}^{n+1}\frac{\Psi_s(\alpha) - \frac{1}{n+1}}{\left|\Psi_s(\alpha) - \frac{1}{n+1}\right|}\left[\pi\left(v_{(s:n)}\,|\alpha\right) - \pi\left(v_{(s-1:n)}\,|\alpha\right)\right] = 0,$$

$$\frac{\partial\eta_1(\alpha)}{\partial\alpha} = \sum_{s=1}^{n+1}\frac{\log\left(\Psi_s(\alpha)\right) - \log\left(\frac{1}{n+1}\right)}{\left|\log\left(\Psi_s(\alpha)\right) - \log\left(\frac{1}{n+1}\right)\right|}\left[\pi\left(v_{(s:n)}\,|\alpha\right) - \pi\left(v_{(s-1:n)}\,|\alpha\right)\right] = 0,$$

$$\frac{\partial\eta_2(\alpha)}{\partial\alpha} = \sum_{s=1}^{n+1}\left[\Psi_s(\alpha) - \left(\frac{1}{n+1}\right)\right]\left[\pi\left(v_{(s:n)}\,|\alpha\right) - \pi\left(v_{(s-1:n)}\,|\alpha\right)\right] = 0,$$

$$\frac{\partial\eta_3(\alpha)}{\partial\alpha} = \sum_{s=1}^{n+1}\left[\log\Psi_s(\alpha) - \log\left(\frac{1}{n+1}\right)\right]\frac{1}{\Psi_s(\alpha)}\left[\pi\left(v_{(s:n)}\,|\alpha\right) - \pi\left(v_{(s-1:n)}\,|\alpha\right)\right] = 0,$$

$$\frac{\partial \eta_4(\alpha)}{\partial \alpha} = \sum_{s=1}^{n+1}\left[e^{\Psi_s(\alpha))-\frac{1}{n+1}} - \left(\Psi_s(\alpha) - \frac{1}{n+1}\right) - 1\right]\left[e^{\Psi_s(\alpha))-\frac{1}{n+1}} - 1\right]\left[\pi\left(v_{(s:n)}\left|\alpha\right.\right) - \pi\left(v_{(s-1:n)}\left|\alpha\right.\right)\right] = 0,$$

where, $\pi\left(v_{(s:n)}\left|\alpha\right.\right)$ and $\pi\left(v_{(s-1:n)}\left|\alpha\right.\right)$ are given in Equations (2.6) and (2.7).

4. Minimum Distance Estimators

Five estimation techniques for the parameter α are demonstrated in this section. They are based on the minimization of the goodness-of-fit statistics with respect to α. The difference between the actual CDF and the estimated CDF is the foundation of this type of statistics.

4.1. Anderson Darling Methods

The AD test, an alternative to other statistical tests for normality, was developed by Anderson and Darling [52]. The AD test is characterized by its quick convergence to the asymptotic distribution, making it a valuable tool for statistical inference ([53, 54]), and recently discussed by several authors [55, 56, 57].

In this subsection, the AD estimator (ADE) $\widehat{\alpha}_8$ of α, the R-AD estimate (R-ADE) $\widehat{\alpha}_9$ of α, the L-AD estimate (L-ADE) $\widehat{\alpha}_{10}$ of α, the AD-LSO estimate (AD-LSOE) $\widehat{\alpha}_{11}$ of α based on the C-JD are produced using the RSS method. Let $V_{(1:n)}, V_{(2:n)}, ..., V_{(n:n)}$ to be an ordered sample forming RSS of size $n = qd$ collected from the C-JD. Then ADE, R-ADE, L-ADE, and AD-LSOE are generated, respectively, by minimizing the following functions:

$$\left.\begin{aligned}
\mathrm{A}(\alpha) &= -n - \frac{1}{n}\sum_{s=1}^{n}(2s-1)\left\{\log H\left(v_{(s:n)}\left|\alpha\right.\right) + \log \bar{H}\left(v_{(n-s+1:n)}\left|\alpha\right.\right)\right\},\\
\mathrm{RT}(\alpha) &= \frac{n}{2} - 2\sum_{s=1}^{n} H\left(v_{(s:n)}\left|\alpha\right.\right) - \frac{1}{n}\sum_{s=1}^{n}(2s-1)\log \bar{H}\left(v_{(n+1-s:n)}\left|\alpha\right.\right),\\
\mathrm{LT}(\alpha) &= \frac{-3n}{2} + 2\sum_{s=1}^{n} H\left(v_{(s:n)}\left|\alpha\right.\right) - \frac{1}{n}\sum_{s=1}^{n}(2s-1)\log H\left(v_{(s:n)}\left|\alpha\right.\right),\\
\mathrm{LO}(\alpha) &= 2\sum_{s=1}^{n}\log\left[H\left(v_{(s:n)}\left|\alpha\right.\right)\right] + \frac{1}{n}\sum_{s=1}^{n}\frac{(2s-1)}{H\left(v_{(s:n)}\left|\alpha\right.\right)},
\end{aligned}\right\}, \tag{4.1}$$

where $\bar{H}\left(.\left|\alpha\right.\right)$ is the survival function of the C-JD. As an alternative to Equation (4.1), the following non-linear equations can be solved numerically after differentiating them with respect to α, to get the estimators $\widehat{\alpha}_8$, $\widehat{\alpha}_9$, $\widehat{\alpha}_{10}$, and $\widehat{\alpha}_{11}$, respectively,

$$\frac{\partial \mathrm{A}(\alpha)}{\partial \alpha} = \sum_{s=1}^{n}(2s-1)\left[\frac{\pi\left(v_{(s:n)}\left|\alpha\right.\right)}{H\left(v_{(s:n)}\left|\alpha\right.\right)} + \frac{\pi\left(v_{(1+n-s:n)}\left|\alpha\right.\right)}{\bar{H}\left(v_{(n-s+1:n)}\left|\alpha\right.\right)}\right] = 0,$$

$$\frac{\partial \mathrm{RT}(\alpha)}{\partial \alpha} = -2\sum_{s=1}^{n}\pi\left(v_{(s:n)}\left|\alpha\right.\right) - \frac{1}{n}\sum_{s=1}^{n}\frac{(2s-1)\pi\left(v_{(n+1-s:n)}\left|\alpha\right.\right)}{\bar{H}\left(v_{(n+1-s:n)}\left|\alpha\right.\right)} = 0,$$

$$\frac{\partial \mathrm{LT}(\alpha)}{\partial \alpha} = 2\sum_{s=1}^{n}\pi\left(v_{(s:n)}\left|\alpha\right.\right) - \frac{1}{n}\sum_{s=1}^{n}\frac{(2s-1)\pi\left(v_{(s:n)}\left|\alpha\right.\right)}{H\left(v_{(s:n)}\left|\alpha\right.\right)} = 0,$$

$$\frac{\partial \mathrm{LO}(\alpha)}{\partial \alpha} = 2\sum_{s=1}^{n} \frac{\pi\left(v_{(s:n)} \,|\alpha\right)}{H\left(v_{(s:n)} \,|\alpha\right)} - \frac{1}{n}\sum_{s=1}^{n} \frac{(2s-1)\pi\left(v_{(s:n)} \,|\alpha\right)}{H^2\left(v_{(s:n)} \,|\alpha\right)} = 0,$$

where, $\pi\left(v_{(s:n)} \,|\alpha\right)$ and $\pi\left(v_{(s-1:n)} \,|\alpha\right)$ are given in Equations (2.6) and (2.7).

4.2. Cramér-von Mises Estimators

Macdonald [58]) showed that the CVM statistic has a lower bias than other minimum distance estimators, offering an alternative approach to parameter estimation. Here, the CVM estimator (CVME) $\widehat{\alpha}_{12}$ of α, is generated based on RSS. Let $V_{(1:n)}, V_{(2:n)}, ..., V_{(n:n)}$ be an ordered sample forming RSS of size $n = qd$, where q is the set size and d is the cycle count, collected from the C-JD. The CVME $\widehat{\alpha}_{12}$ of α, is determined by minimizing the following function:

$$\omega(\alpha) = \frac{1}{12n} + \sum_{s=1}^{n} \left\{ H\left(v_{(s:n)} \,|\lambda\right) - \frac{2s-1}{2n} \right\}^2. \tag{4.2}$$

In lieu of using Equation (4.2), the non-linear equation below may be solved to obtain $\widehat{\alpha}_{12}$

$$\frac{\partial \omega(\alpha)}{\partial \alpha} = \sum_{s=1}^{n} \left\{ H\left(v_{(s:n)} \,|\lambda\right) - \frac{2s-1}{2n} \right\} \pi\left(v_{(s:n)} \,|\lambda\right) = 0,$$

where, $\pi\left(v_{(s:n)} \,|\alpha\right)$ and $\pi\left(v_{(s-1:n)} \,|\alpha\right)$ are given in Equations (2.6) and (2.7).

5. Additional Estimators

Swain et al. [59] presented the OLS estimate (OLSE) and WLSE estimate (WLSE) to estimate the beta distribution's parameters. This section uses RSS to construct the OLSE, WLSE, and Kolmogorov estimate (KE) of the C-JD. Let $V_{(1:n)}, V_{(2:n)}, ..., V_{(n:n)}$ to be an ordered sample forming RSS of size $n = qd$ collected from the C-JD. Upon minimizing the following functions concerning α, the OLSE $\widehat{\alpha}_{13}$ and WLSE $\widehat{\alpha}_{14}$ are gained.

$$\varpi(\alpha) = \sum_{s=1}^{n} \left[H\left(v_{(s:n)} \,|\alpha\right) - \frac{s}{n+1} \right]^2, \tag{5.1}$$

and,

$$\varpi_1(\alpha) = \sum_{s=1}^{n} \frac{(n+1)^2(n+2)}{s(n-s+1)} \left[H\left(v_{(s:n)} \,|\alpha\right) - \frac{s}{n+1} \right]^2. \tag{5.2}$$

As an alternative to minimizing Equations (5.1) and (5.2), the OLSE $\widehat{\alpha}_{13}$ and WLSE $\widehat{\alpha}_{14}$ can be obtained by solving the following non-linear equations

$$\frac{\partial \varpi(\alpha)}{\partial \alpha} = \sum_{s=1}^{n} \left[H\left(v_{(s:n)} \,|\alpha\right) - \frac{s}{n+1} \right] \pi\left(v_{(s:n)} \,|\alpha\right) = 0,$$

$$\frac{\partial \varpi_1(\alpha)}{\partial \alpha} = \sum_{s=1}^{n} \frac{(n+1)^2(n+2)}{s(n-s+1)} \left[H\left(v_{(s:n)} \,|\alpha\right) - \frac{s}{n+1} \right] \pi\left(v_{(s:n)} \,|\alpha\right) = 0,$$

where, $\pi\left(v_{(s:n)}\,|\alpha\right)$ in Equation (2.6).
After that, given $V_{(1:n)}, V_{(2:n)}, ..., V_{(n:n)}$ be an ordered sample forming RSS of size $n = qd$ collected from the C-JD. The following function is minimized, with respect to α, to give the KE $\widehat{\alpha}_{15}$

$$\varpi_2(\alpha) = \underset{1\leqslant s\leqslant n}{Max} \sum_{s=1}^{n} \left[\frac{s}{n} - \{[1 - \mathrm{A}(v_{s:n}, \alpha)]\}, \{1 - [1 - \mathrm{A}(v_{s:n}, \alpha)]\} - \frac{s-1}{n}\right]^2.$$

6. Numerical Simulation

This section evaluates the effectiveness of several estimating methods for the C-JD. Random datasets will be generated and prioritized, and various methodologies will be used to determine the most suited. In addition, the datasets will be rated, and the optimal alternative will be determined using estimation techniques. The simulation will be run through the following steps:

1. Generate SRS of sizes $n = 20, 60, 120, 200, 300,$ and 400 from the C-JD.
2. Create RSS from the C-JD with set size $q = 5$ and cycle numbers $d = 4, 12, 24, 40, 60,$ and 80, where $n = qd$.
3. Determine the C-JD estimates $\hat{\alpha}$ for the two generation methods (RSS and SRS) thousands of times.
4. Six different measures are utilized to evaluate the estimation methods, delineated as follows:
   - The absolute bias, $|E_1(\widehat{\boldsymbol{\alpha}})| = \frac{1}{M}\sum_{i=1}^{M}|\widehat{\boldsymbol{\alpha}} - \boldsymbol{\alpha}|$.
   - The mean squared error, $E_2 = \frac{1}{M}\sum_{i=1}^{M}(\widehat{\boldsymbol{\alpha}} - \boldsymbol{\alpha})^2$.
   - The mean absolute relative error, $E_3 = \frac{1}{M}\sum_{i=1}^{M}|\widehat{\boldsymbol{\alpha}} - \boldsymbol{\alpha}|/\boldsymbol{\alpha}$.
   - The average absolute difference, $E_4 = \frac{1}{nH}\sum_{i=1}^{H}\sum_{j=1}^{n}|F(v_{ij};\boldsymbol{\alpha}) - F(v_{ij};\widehat{\boldsymbol{\alpha}})|$, where $F(v;\boldsymbol{\alpha}) = F(v)$ and v_{ij} represents values obtained at the i-th iteration sample and j-th component of this sample.
   - The maximum absolute difference, $E_5 = \frac{1}{H}\sum_{i=1}^{H}\max_{j=1,\dots,n}|F(v_{ij};\boldsymbol{\alpha}) - F(v_{ij};\widehat{\boldsymbol{\alpha}})|$.
   - The average squared absolute error, $E_6 = \frac{1}{M}\sum_{i=1}^{M}\frac{|v_{(i)} - \hat{v}(i)|}{v_{(n)} - v_{(1)}}$, $v_{(i)}$ is the ascending ordered observations.
5. Tables 1 to 10 provide a complete summary of the findings and allow for estimating approach comparisons.
6. Table 11 shows the ratio of SRS to RSS MSE, allowing for a comparison of sampling strategies' performance.
7. Tables 12 and 13 provide a detailed study of the performance of SRS and RSS estimations, including partial and total ranks.

Upon meticulous scrutiny of the simulation outcomes and rankings, the following results have been deduced:

- Model estimates are consistent across both the SRS and RSS datasets, indicating that they will approach real parameter values as sample size rises.
- As sample size rises, all metrics steadily decline, suggesting that bigger samples provide better accuracy.

Table 1. Numerical values for simulation measures for ($\alpha = 0.3$) under SRS.

n		MLE	ADE	CVME	MPSE	OLSE	R-ADE	WLSE	L-ADE	MNSADE	MNSALODE	AD-LSOE	KE	MSSD	MNSSLOD	MNSLXD
20	E_1	0.03344[4]	0.03409[6]	0.03652[9]	0.03303[2]	0.03615[7]	0.03221[1]	0.03311[3]	0.03878[11]	0.04079[12]	0.0364[8]	0.0564[15]	0.03697[10]	0.04393[14]	0.03384[5]	0.04302[13]
	E_2	0.00206[6]	0.00196[5]	0.00217[8]	0.00175[2]	0.00225[10]	0.00173[1]	0.0018[3]	0.00257[11]	0.00291[12]	0.00207[7]	0.00587[15]	0.00223[9]	0.00319[14]	0.00181[4]	0.00298[13]
	E_3	0.11147[4]	0.11364[6]	0.12172[9]	0.11011[2]	0.1205[7]	0.10738[1]	0.11038[3]	0.12926[11]	0.13595[12]	0.12134[8]	0.188[15]	0.12323[10]	0.14642[14]	0.1128[5]	0.14341[13]
	E_4	0.04549[1]	0.04957[5]	0.05296[8]	0.04946[4]	0.05241[7]	0.04744[2]	0.04868[3]	0.05619[11]	0.06002[12]	0.05521[10]	0.07491[15]	0.05367[9]	0.06462[14]	0.05049[6]	0.06409[13]
	E_5	0.066[1]	0.07149[4]	0.07699[8]	0.07164[5]	0.07606[7]	0.06843[2]	0.07023[3]	0.08133[11]	0.08673[12]	0.07908[10]	0.11051[15]	0.07751[9]	0.09489[14]	0.07325[6]	0.09342[13]
	E_6	0.05187[4]	0.05138[3]	0.05224[6]	0.05197[5]	0.05328[7]	0.04969[1]	0.05094[2]	0.05653[10]	0.06112[12]	0.05849[11]	0.07386[15]	0.05382[8]	0.06548[13]	0.05592[9]	0.06573[14]
	$\sum$ *Ranks*	20[3.5]	29[5]	48[8]	20[3.5]	45[7]	8[1]	17[2]	65[11]	72[12]	54[9]	90[15]	55[10]	83[14]	35[6]	79[13]
60	E_1	0.01928[4]	0.01917[3]	0.02094[8]	0.01898[2]	0.02057[7]	0.01889[1]	0.01993[5]	0.02269[11]	0.02377[12]	0.0211[9]	0.03719[15]	0.02016[6]	0.02511[13]	0.02113[10]	0.02586[14]
	E_2	0.00061[4]	0.00059[3]	$7e-04$[8.5]	0.00057[1.5]	0.00068[7]	0.00057[1.5]	0.00063[5]	0.00082[11]	0.00089[12]	0.00072[10]	0.00274[15]	0.00065[6]	0.00104[13]	$7e-04$[8.5]	0.00109[14]
	E_3	0.06426[4]	0.06391[3]	0.0698[8]	0.06328[2]	0.06856[7]	0.06296[1]	0.06642[5]	0.07562[11]	0.07922[12]	0.07034[9]	0.12397[15]	0.06721[6]	0.08369[13]	0.07042[10]	0.0862[14]
	E_4	0.02761[1]	0.02828[3]	0.03054[8]	0.02835[4]	0.02992[7]	0.02782[2]	0.02911[5]	0.03301[11]	0.03519[12]	0.03155[10]	0.05047[15]	0.02967[6]	0.03722[13]	0.03153[9]	0.03803[14]
	E_5	0.04048[1]	0.04124[3]	0.04466[8]	0.0413[4]	0.04385[7]	0.04055[2]	0.04259[5]	0.04843[11]	0.05144[12]	0.0459[9]	0.07471[15]	0.04331[6]	0.05426[13]	0.04609[10]	0.05566[14]
	E_6	0.02486[3]	0.02474[2]	0.02565[7]	0.025[4]	0.02563[6]	0.02472[1]	0.02507[5]	0.02779[10]	0.03085[12]	0.02855[11]	0.03953[15]	0.0268[8]	0.03281[13]	0.02771[9]	0.03322[14]
	$\sum$ *Ranks*	17[2.5]	17[2.5]	53.5[8]	17.5[4]	41[7]	8.5[1]	30[5]	63[11]	70[12]	56[9]	88[15]	38[6]	76[13]	62.5[10]	82[14]
120	E_1	0.01286[1]	0.01385[5]	0.01447[7]	0.01315[2]	0.01445[6]	0.01328[3]	0.0138[4]	0.01576[11]	0.01704[12]	0.01528[10]	0.02666[15]	0.01452[9]	0.01821[13]	0.01449[8]	0.01846[14]
	E_2	0.00027[1.5]	0.00031[5]	0.00034[8.5]	0.00027[1.5]	0.00033[7]	0.00028[3]	$3e-04$[4]	$4e-04$[11]	0.00046[12]	0.00037[10]	0.00148[15]	0.00034[8.5]	0.00052[13]	0.00032[6]	0.00053[14]
	E_3	0.04288[1]	0.04616[5]	0.04823[7]	0.04384[2]	0.04817[6]	0.04427[3]	0.04601[4]	0.05252[11]	0.05679[12]	0.05092[10]	0.08885[15]	0.04839[9]	0.0607[13]	0.04829[8]	0.06153[14]
	E_4	0.01892[1]	0.02046[5]	0.02125[6]	0.01966[2]	0.02131[7]	0.01967[3]	0.02044[4]	0.02311[11]	0.02517[12]	0.02275[10]	0.03656[15]	0.02139[8]	0.02703[13]	0.02149[9]	0.0273[14]
	E_5	0.02764[1]	0.02987[5]	0.03104[6]	0.02867[3]	0.03115[7]	0.02865[2]	0.02986[4]	0.03387[11]	0.03687[12]	0.03325[10]	0.05423[15]	0.0312[8]	0.0396[13]	0.03148[9]	0.04003[14]
	E_6	0.01625[3]	0.01644[5]	0.01687[7]	0.01636[4]	0.01623[2]	0.01605[1]	0.01658[6]	0.01862[10]	0.02045[12]	0.01888[11]	0.02692[15]	0.01747[8]	0.02151[14]	0.01828[9]	0.02141[13]
	$\sum$ *Ranks*	8.5[1]	29[4]	40.5[7]	14.5[2]	34[5]	15[3]	36[6]	69[11]	70[12]	60[10]	88[15]	49.5[9]	77[13]	48[8]	81[14]
200	E_1	0.01008[1]	0.01069[5]	0.01147[9]	0.01042[2]	0.01101[6]	0.01055[3]	0.01063[4]	0.01172[10]	0.01312[12]	0.01198[11]	0.01962[15]	0.01109[7.5]	0.01373[13]	0.01109[7.5]	0.01406[14]
	E_2	0.00016[1]	0.00018[4]	0.00021[9.5]	0.00017[2.5]	0.00019[6.5]	0.00017[2.5]	0.00019[6.5]	0.00021[9.5]	0.00026[12]	0.00023[11]	0.00078[15]	0.00019[6.5]	0.00031[13]	0.00019[6.5]	0.00032[14]
	E_3	0.03359[1]	0.03563[5]	0.03822[9]	0.03474[2]	0.03669[6]	0.03515[3]	0.03543[4]	0.03907[10]	0.04374[12]	0.03992[11]	0.06539[15]	0.03695[7]	0.04578[13]	0.03697[8]	0.04688[14]
	E_4	0.0149[1]	0.01585[5]	0.01689[9]	0.01549[2]	0.0163[6]	0.01559[3]	0.01565[4]	0.01725[10]	0.01956[12]	0.01778[11]	0.02759[15]	0.01639[7]	0.02036[13]	0.01646[8]	0.02089[14]
	E_5	0.02179[1]	0.0231[5]	0.02473[9]	0.02262[2]	0.02378[6]	0.0228[3]	0.02287[4]	0.02526[10]	0.02858[12]	0.02597[11]	0.04073[15]	0.02393[7]	0.02981[13]	0.02408[8]	0.03048[14]
	E_6	0.01193[4]	0.01164[1]	0.01236[7]	0.01197[5]	0.01199[6]	0.0118[3]	0.01175[2]	0.01302[9]	0.01464[12]	0.01378[11]	0.01946[15]	0.0127[8]	0.01541[13]	0.01324[10]	0.01584[14]
	$\sum$ *Ranks*	9[1]	25[5]	52.5[9]	15.5[2]	36.5[6]	17.5[3]	24.5[4]	58.5[10]	72[12]	66[11]	90[15]	43[7]	78[13]	48[8]	84[14]
300	E_1	0.00796[1]	0.00837[2]	0.00886[7]	0.00841[3]	0.00868[6]	0.00867[5]	0.00842[4]	0.00946[9]	0.01091[12]	0.00966[11]	0.01567[15]	0.0093[8]	0.01135[13]	0.00949[10]	0.0115[14]
	E_2	$1e-04$[1]	0.00011[3]	0.00013[7.5]	0.00011[3]	0.00012[5.5]	0.00012[5.5]	0.00011[3]	0.00015[11]	0.00019[12]	0.00014[9.5]	0.00047[15]	0.00013[7.5]	$2e-04$[13]	0.00014[9.5]	0.00021[14]
	E_3	0.02653[1]	0.02789[2]	0.02955[7]	0.02803[3]	0.02895[6]	0.02891[5]	0.02807[4]	0.03153[9]	0.03636[12]	0.03222[11]	0.05223[15]	0.03101[8]	0.03784[13]	0.03163[10]	0.03832[14]
	E_4	0.01176[1]	0.01236[2]	0.01311[7]	0.01253[4]	0.01281[5]	0.01284[6]	0.01242[3]	0.01397[9]	0.01614[12]	0.01432[11]	0.02241[15]	0.01374[8]	0.01688[13]	0.01411[10]	0.01706[14]
	E_5	0.0172[1]	0.01808[2]	0.01918[7]	0.01828[4]	0.01875[5]	0.01877[6]	0.01816[3]	0.02041[9]	0.0236[12]	0.02094[11]	0.03294[15]	0.02008[8]	0.02467[13]	0.02061[10]	0.02493[14]
	E_6	0.00955[6]	0.00943[4]	0.00953[5]	0.00928[3]	0.00963[7]	0.00913[1]	0.00919[2]	0.0104[9.5]	0.01167[12]	0.01071[11]	0.01511[15]	0.00989[8]	0.01218[13]	0.0104[9.5]	0.01242[14]
	$\sum$ *Ranks*	24[4]	14[1]	39.5[7]	19[3]	33.5[6]	27.5[5]	18[2]	55.5[9]	71[12]	63.5[11]	88[15]	46.5[8]	80[13]	58[10]	82[14]
400	E_1	0.00704[2]	0.00755[5]	0.00793[9]	0.00712[3]	0.00769[8]	0.00686[1]	0.00748[4]	0.00839[11]	0.00865[12]	0.0081[10]	0.01239[15]	0.00763[7]	0.01005[14]	0.0076[6]	0.00984[13]
	E_2	$8e-05$[2]	$9e-05$[5.5]	$1e-04$[9]	$8e-05$[2]	$1e-04$[9]	$8e-05$[2]	$9e-05$[5.5]	0.00011[11]	0.00012[12]	$1e-04$[9]	0.00027[15]	$9e-05$[5.5]	0.00016[14]	$9e-05$[5.5]	0.00015[13]
	E_3	0.02346[2]	0.02516[5]	0.02644[9]	0.02372[3]	0.02563[8]	0.02288[1]	0.02492[4]	0.02797[11]	0.02884[12]	0.02701[10]	0.04129[15]	0.02544[7]	0.0335[14]	0.02532[6]	0.0328[13]
	E_4	0.0104[2]	0.01114[5]	0.01172[9]	0.01055[3]	0.01133[8]	0.01016[1]	0.01104[4]	0.01241[11]	0.01285[12]	0.012[10]	0.01793[15]	0.01131[7]	0.01494[14]	0.01127[6]	0.01455[13]
	E_5	0.01522[2]	0.01631[5]	0.01715[9]	0.01544[3]	0.01658[8]	0.01485[1]	0.01616[4]	0.01815[11]	0.01875[12]	0.01754[10]	0.02634[15]	0.01651[7]	0.02185[14]	0.01646[6]	0.0213[13]
	E_6	0.0079[5]	0.00789[4]	0.00805[7]	0.00773[2]	0.00798[6]	0.00771[1]	0.00782[3]	0.00857[9]	0.00969[12]	0.00908[11]	0.01219[15]	0.0084[8]	0.01057[14]	0.00871[10]	0.01029[13]
	$\sum$ *Ranks*	23[2]	37.5[5]	50[9]	24[3]	45[6]	15[1]	32.5[4]	54[10]	62[12]	58[11]	80[15]	49.5[8]	74[14]	47.5[7]	68[13]

Table 2. Numerical values for simulation measures for ($\alpha = 0.3$) under RSS.

n		MLE	ADE	CVME	MPSE	OLSE	R-ADE	WLSE	L-ADE	MNSADE	MNSALODE	AD-LSOE	KE	MSSD	MNSSLOD	MNSLXD
20	E_1	0.01946[1]	0.0199[2]	0.02059[5]	0.02156[7]	0.02075[6]	0.02029[4]	0.02023[3]	0.02184[8]	0.03114[12]	0.02817[11]	0.04478[15]	0.0221[9]	0.03346[13]	0.02406[10]	0.03421[14]
	E_2	0.00062[2]	0.00061[1]	0.00069[6]	0.00072[7]	0.00066[5]	0.00064[4]	0.00063[3]	0.00079[8]	0.00153[12]	0.0012[11]	0.00443[15]	8e − 04[9]	0.00189[13]	0.00096[10]	0.00205[14]
	E_3	0.06486[1]	0.06633[2]	0.06862[5]	0.07186[7]	0.06918[6]	0.06764[4]	0.06742[3]	0.07279[8]	0.1038[12]	0.09389[11]	0.14928[15]	0.07365[9]	0.11155[13]	0.08021[10]	0.11404[14]
	E_4	0.0282[1]	0.02946[2]	0.0301[4]	0.03264[9]	0.03045[6]	0.03035[5]	0.02999[3]	0.03186[7]	0.04669[12]	0.04292[11]	0.05792[15]	0.0325[8]	0.04983[13]	0.03599[10]	0.05086[14]
	E_5	0.04137[1]	0.0429[2]	0.04416[5]	0.04784[9]	0.04462[6]	0.04414[4]	0.04356[3]	0.04653[7]	0.06818[12]	0.06243[11]	0.08564[15]	0.04722[8]	0.07334[13]	0.05272[10]	0.07398[14]
	E_6	0.04803[4]	0.04752[3]	0.04922[7]	0.04865[6]	0.04819[5]	0.04702[2]	0.04697[1]	0.05088[9]	0.05817[12]	0.05555[11]	0.07159[15]	0.05063[8]	0.06355[13]	0.05327[10]	0.06386[14]
	$\sum$ *Ranks*	10[1]	12[2]	32[5]	45[7]	34[6]	23[4]	16[3]	47[8]	71[12]	65[11]	89[15]	57[9]	77[13]	59[10]	83[14]
60	E_1	0.01099[2]	0.01095[1]	0.01175[4]	0.01198[7]	0.01179[5]	0.01186[6]	0.01127[3]	0.01283[9]	0.01859[12]	0.01631[11]	0.02782[15]	0.01208[8]	0.01976[13]	0.01513[10]	0.02022[14]
	E_2	0.00019[1.5]	0.00019[1.5]	0.00022[4.5]	0.00022[4.5]	0.00023[7]	0.00023[7]	2e − 04[3]	0.00025[9]	0.00056[12]	0.00042[11]	0.00169[15]	0.00023[7]	0.00066[13.5]	0.00037[10]	0.00066[13.5]
	E_3	0.03663[2]	0.03649[1]	0.03917[4]	0.03993[7]	0.0393[5]	0.03955[6]	0.03756[3]	0.04276[9]	0.06198[12]	0.05435[11]	0.09272[15]	0.04026[8]	0.06585[13]	0.05045[10]	0.0674[14]
	E_4	0.01613[1]	0.01618[2]	0.01738[5]	0.01788[8]	0.01737[4]	0.01757[6]	0.01665[3]	0.01882[9]	0.02769[12]	0.02447[11]	0.0379[15]	0.01771[7]	0.02947[13]	0.02263[10]	0.03027[14]
	E_5	0.02373[2]	0.02366[1]	0.02536[4]	0.02618[8]	0.02542[5]	0.0257[6]	0.02439[3]	0.02758[9]	0.04057[12]	0.03573[11]	0.0559[15]	0.02599[7]	0.04313[13]	0.03311[10]	0.04433[14]
	E_6	0.02402[5]	0.02358[3]	0.0241[6]	0.02372[4]	0.02434[7]	0.02324[1]	0.02345[2]	0.02555[9]	0.02952[12]	0.02808[11]	0.03727[15]	0.02515[8]	0.03101[13]	0.02642[10]	0.03113[14]
	$\sum$ *Ranks*	13.5[2]	9.5[1]	26.5[3]	37.5[7]	32[6]	31[5]	29[4]	53[9]	71[12]	65[11]	89[15]	44[8]	77.5[13]	59[10]	82.5[14]
120	E_1	0.00784[1]	0.00833[4]	0.00844[5]	0.00886[8]	0.00853[6]	0.00788[2]	0.00796[3]	0.00938[9]	0.01345[12]	0.01149[11]	0.02029[15]	0.00866[7]	0.01462[13]	0.01062[10]	0.01476[14]
	E_2	1e − 04[2]	0.00011[5]	0.00011[5]	0.00012[7.5]	0.00011[5]	1e − 04[2]	1e − 04[2]	0.00014[9]	0.00028[12]	2e − 04[11]	0.00105[15]	0.00012[7.5]	0.00034[13]	0.00018[10]	0.00035[14]
	E_3	0.02613[1]	0.02778[4]	0.02815[5]	0.02953[8]	0.02843[6]	0.02627[2]	0.02655[3]	0.03128[9]	0.04483[12]	0.03829[11]	0.06764[15]	0.02886[7]	0.04874[13]	0.03541[10]	0.04919[14]
	E_4	0.01159[1]	0.01233[4]	0.01248[5]	0.01322[8]	0.0126[6]	0.01168[2]	0.01174[3]	0.01382[9]	0.01991[12]	0.01711[11]	0.02798[15]	0.01277[7]	0.02182[13]	0.01577[10]	0.02193[14]
	E_5	0.01697[1]	0.01801[4]	0.01823[5]	0.01932[8]	0.01845[6]	0.01702[2]	0.01718[3]	0.02023[9]	0.02919[12]	0.02509[11]	0.04119[15]	0.01868[7]	0.03195[13]	0.02314[10]	0.03214[14]
	E_6	0.01519[1]	0.01524[2]	0.0156[6.5]	0.01547[5]	0.0156[6.5]	0.01533[3]	0.01546[4]	0.01649[9]	0.0195[12]	0.0181[11]	0.02527[15]	0.01624[8]	0.02094[14]	0.01728[10]	0.02041[13]
	$\sum$ *Ranks*	18[1]	20[2]	28.5[4]	41.5[8]	32.5[6]	24[3]	29[5]	51[9]	68[11]	70[12]	86[15]	40.5[7]	75[13]	57[10]	79[14]
200	E_1	0.00628[3]	0.00595[1]	0.00635[4]	0.0065[6]	0.00661[7]	0.00625[2]	0.00642[5]	0.0071[9]	0.0103[12]	0.00914[11]	0.01442[15]	0.00688[8]	0.01163[14]	0.00846[10]	0.01156[13]
	E_2	6e − 05[3]	6e − 05[3]	6e − 05[3]	7e − 05[7]	7e − 05[7]	6e − 05[3]	6e − 05[3]	8e − 05[9]	0.00017[12]	0.00013[11]	0.00049[15]	7e − 05[7]	0.00021[13.5]	0.00011[10]	0.00021[13.5]
	E_3	0.02092[3]	0.01985[1]	0.02116[4]	0.02168[6]	0.02204[7]	0.02084[2]	0.02139[5]	0.02366[9]	0.03432[12]	0.03047[11]	0.04808[15]	0.02295[8]	0.03876[14]	0.0282[10]	0.03853[13]
	E_4	0.00927[3]	0.00882[1]	0.00937[4]	0.00968[6]	0.00978[7]	0.00926[2]	0.00948[5]	0.01047[9]	0.01528[12]	0.01363[11]	0.02044[15]	0.01021[8]	0.01729[14]	0.01259[10]	0.01721[13]
	E_5	0.01356[3]	0.01288[1]	0.01372[4]	0.01415[6]	0.0143[7]	0.01355[2]	0.01388[5]	0.01532[9]	0.02235[12]	0.01989[11]	0.02997[15]	0.01489[8]	0.02529[14]	0.01841[10]	0.02515[13]
	E_6	0.01133[4]	0.01128[3]	0.01161[6]	0.01138[5]	0.01169[7]	0.01111[1]	0.01113[2]	0.01191[9]	0.01407[12]	0.01317[11]	0.01738[15]	0.01182[8]	0.01531[14]	0.01277[10]	0.01529[13]
	$\sum$ *Ranks*	25[3]	16[1]	31[4.5]	42[6]	48[7]	18[2]	31[4.5]	60[11]	63[12]	57[10]	81[15]	53[9]	74.5[14]	51[8]	69.5[13]
300	E_1	0.00483[1]	0.00497[2]	0.00512[4]	0.00545[7]	0.00516[5]	0.00533[6]	0.00504[3]	0.00584[9]	0.00837[12]	0.00726[11]	0.01213[15]	0.00566[8]	0.00948[13]	0.00697[10]	0.00968[14]
	E_2	4e − 05[3.5]	4e − 05[3.5]	4e − 05[3.5]	5e − 05[8]	4e − 05[3.5]	4e − 05[3.5]	4e − 05[3.5]	5e − 05[8]	0.00011[12]	8e − 05[11]	0.00036[15]	5e − 05[8]	0.00014[13.5]	7e − 05[10]	0.00014[13.5]
	E_3	0.01611[1]	0.01658[2]	0.01705[4]	0.01815[7]	0.01719[5]	0.01777[6]	0.0168[3]	0.01947[9]	0.02791[12]	0.02419[11]	0.04043[15]	0.01886[8]	0.03159[13]	0.02325[10]	0.03225[14]
	E_4	0.00714[1]	0.00735[2]	0.00758[4]	0.0081[7]	0.00762[5]	0.0079[6]	0.00744[3]	0.00865[9]	0.01238[12]	0.01075[11]	0.0173[15]	0.00837[8]	0.0141[13]	0.01036[10]	0.01442[14]
	E_5	0.01045[1]	0.01077[2]	0.01107[4]	0.01184[7]	0.01115[5]	0.01154[6]	0.0109[3]	0.01263[9]	0.01815[12]	0.01573[11]	0.02533[15]	0.01224[8]	0.0206[13]	0.01515[10]	0.02107[14]
	E_6	0.00874[2]	0.00888[4]	0.00891[6]	0.00889[5]	0.00896[7]	0.00868[1]	0.0088[3]	0.00958[9]	0.01106[12]	0.01035[11]	0.01407[15]	0.00935[8]	0.01189[13]	0.00987[10]	0.01209[14]
	$\sum$ *Ranks*	13.5[1]	19.5[2]	29.5[4]	45[7]	34.5[6]	32.5[5]	22.5[3]	57[9]	61[10]	70[13]	79[15]	52[8]	67.5[12]	64[11]	72.5[14]
400	E_1	0.00433[3]	0.00426[2]	0.00412[1]	0.00466[7]	0.0045[6]	0.00449[5]	0.00443[4]	0.00517[9]	0.00727[12]	0.00637[11]	0.01043[15]	0.00488[8]	0.00853[14]	0.00584[10]	0.00817[13]
	E_2	3e − 05[4]	3e − 05[4]	3e − 05[4]	3e − 05[4]	3e − 05[4]	3e − 05[4]	3e − 05[4]	4e − 05[8.5]	8e − 05[12]	6e − 05[10.5]	0.00025[15]	4e − 05[8.5]	0.00011[13.5]	6e − 05[10.5]	0.00011[13.5]
	E_3	0.01442[3]	0.01421[2]	0.01373[1]	0.01555[7]	0.015[6]	0.01495[5]	0.01477[4]	0.01724[9]	0.02423[12]	0.02124[11]	0.03477[15]	0.01627[8]	0.02843[14]	0.01946[10]	0.02725[13]
	E_4	0.00641[3]	0.0063[2]	0.00608[1]	0.00691[7]	0.00664[5.5]	0.00664[5.5]	0.00654[4]	0.00763[9]	0.01079[12]	0.00943[11]	0.015[15]	0.00721[8]	0.01263[14]	0.00868[10]	0.01212[13]
	E_5	0.00936[3]	0.00922[2]	0.0089[1]	0.01012[7]	0.00973[6]	0.00971[5]	0.00958[4]	0.01117[9]	0.01575[12]	0.01382[11]	0.022[15]	0.01054[8]	0.01848[14]	0.01268[10]	0.01774[13]
	E_6	0.00729[1]	0.00747[4]	0.0075[5.5]	0.00743[3]	0.00752[7]	0.00734[2]	0.0075[5.5]	0.00789[9]	0.00929[12]	0.00867[11]	0.01164[15]	0.00785[8]	0.01008[13]	0.00833[10]	0.0101[14]
	$\sum$ *Ranks*	20[3]	19[2]	16.5[1]	38[7]	37.5[6]	29.5[5]	28.5[4]	56.5[9]	75[14]	68.5[12]	78[15]	51.5[8]	70.5[13]	63.5[10]	67.5[11]

Table 3. Numerical values for simulation measures for ($\alpha = 0.9$) under SRS.

n		MLE	ADE	CVME	MPSE	OLSE	R-ADE	WLSE	L-ADE	MNSADE	MNSALODE	AD-LSOE	KE	MSSD	MNSSLOD	MNSLXD
20	E_1	0.10407[2]	0.10961[4]	0.11728[8]	0.10222[1]	0.11817[9]	0.10648[3]	0.11313[6]	0.13282[12]	0.13292[13]	0.11494[7]	0.17483[15]	0.12068[10]	0.13256[11]	0.11102[5]	0.14393[14]
	E_2	0.0183[3]	0.01934[4]	0.02398[8]	0.01632[1]	0.02482[9]	0.0177[2]	0.02067[6]	0.03248[13]	0.02969[12]	0.02103[7]	0.05621[15]	0.02531[10]	0.02894[11]	0.01953[5]	0.03511[14]
	E_3	0.11563[2]	0.12178[4]	0.13031[8]	0.11358[1]	0.13129[9]	0.11831[3]	0.12571[6]	0.14757[12]	0.14769[13]	0.12771[7]	0.19426[15]	0.13409[10]	0.14729[11]	0.12336[5]	0.15993[14]
	E_4	0.0451[1]	0.04925[4]	0.05135[7]	0.04744[2]	0.05226[8]	0.04837[3]	0.05086[6]	0.05615[11]	0.06077[12]	0.05334[10]	0.07162[15]	0.0528[9]	0.06133[13]	0.05018[5]	0.06514[14]
	E_5	0.0671[1]	0.07208[4]	0.07574[7]	0.06933[2]	0.07676[8]	0.0709[3]	0.07458[6]	0.08357[11]	0.0894[12]	0.0786[10]	0.1064[15]	0.07773[9]	0.09095[13]	0.07415[5]	0.09671[14]
	E_6	0.05177[5]	0.05007[2]	0.05342[7]	0.05048[3]	0.05258[6]	0.04907[1]	0.0512[4]	0.05754[10]	0.06247[12]	0.05812[11]	0.07591[15]	0.05512[9]	0.06691[14]	0.05511[8]	0.06613[13]
	$\sum$ *Ranks*	14[2]	22[4]	45[7]	10[1]	49[8]	15[3]	34[6]	69[11]	74[13]	52[9]	90[15]	57[10]	73[12]	33[5]	83[14]
60	E_1	0.06081[2]	0.06467[6]	0.06405[5]	0.05816[1]	0.06599[7]	0.06252[3]	0.06379[4]	0.07056[10]	0.0764[12]	0.06635[8]	0.11833[15]	0.072[11]	0.08158[13]	0.07024[9]	0.08661[14]
	E_2	0.00602[2]	0.00649[5.5]	0.00649[5.5]	0.00529[1]	0.00715[8]	0.00621[3]	0.00647[4]	0.00843[11]	0.00953[12]	0.00679[7]	0.0273[15]	0.0082[10]	0.01051[13]	0.00771[9]	0.01259[14]
	E_3	0.06757[2]	0.07185[6]	0.07117[5]	0.06462[1]	0.07332[7]	0.06946[3]	0.07088[4]	0.0784[10]	0.08489[12]	0.07373[8]	0.13147[15]	0.08[11]	0.09064[13]	0.07804[9]	0.09624[14]
	E_4	0.02722[2]	0.02905[6]	0.02897[5]	0.02667[1]	0.02981[7]	0.02815[3]	0.02867[4]	0.03148[9]	0.03461[12]	0.03052[8]	0.04923[15]	0.03224[11]	0.03719[13]	0.03205[10]	0.03918[14]
	E_5	0.04029[2]	0.04304[6]	0.04281[5]	0.03947[1]	0.04424[7]	0.04175[3]	0.04251[4]	0.04677[9]	0.05133[12]	0.04505[8]	0.07378[15]	0.0478[11]	0.05536[13]	0.04734[10]	0.0581[14]
	E_6	0.025[3]	0.02475[1]	0.02552[5]	0.02499[2]	0.02687[7]	0.02508[4]	0.02573[6]	0.02797[10]	0.03171[12]	0.0288[11]	0.04176[15]	0.0275[9]	0.03336[13]	0.02742[8]	0.03464[14]
	$\sum$ *Ranks*	13[2]	30.5[5.5]	30.5[5.5]	7[1]	43[7]	19[3]	26[4]	59[10]	72[12]	50[8]	90[15]	63[11]	78[13]	55[9]	84[14]
120	E_1	0.04061[1]	0.04507[5]	0.04624[6]	0.04205[2]	0.04629[7]	0.04406[3]	0.04467[4]	0.05285[11]	0.05382[12]	0.04811[10]	0.08653[15]	0.04778[9]	0.05879[13]	0.04687[8]	0.06011[14]
	E_2	0.00266[1]	0.0032[5]	0.00329[6]	0.00279[2]	0.00337[7]	0.00309[3]	0.0031[4]	0.00434[11]	0.00453[12]	0.0036[9]	0.01439[15]	0.00373[10]	0.00535[13]	0.00341[8]	0.00581[14]
	E_3	0.04512[1]	0.05008[5]	0.05138[6]	0.04672[2]	0.05143[7]	0.04895[3]	0.04963[4]	0.05872[11]	0.0598[12]	0.05346[10]	0.09614[15]	0.05309[9]	0.06532[13]	0.05207[8]	0.06679[14]
	E_4	0.01829[1]	0.02033[5]	0.02084[6]	0.01921[2]	0.021[7]	0.0198[3]	0.02021[4]	0.02378[11]	0.02444[12]	0.02201[10]	0.03653[15]	0.0214[9]	0.02681[13]	0.02138[8]	0.02733[14]
	E_5	0.0272[1]	0.0302[5]	0.03094[6]	0.02843[2]	0.03108[7]	0.0294[3]	0.02993[4]	0.03533[11]	0.03627[12]	0.03261[10]	0.05486[15]	0.03183[9]	0.03976[13]	0.0317[8]	0.04066[14]
	E_6	0.01659[5]	0.01658[4]	0.01702[6]	0.01651[3]	0.01708[7]	0.0162[1]	0.0163[2]	0.01858[10]	0.02071[12]	0.01883[11]	0.0279[15]	0.01806[8]	0.02183[13]	0.01827[9]	0.02211[14]
	$\sum$ *Ranks*	10[1]	29[5]	36[6]	13[2]	42[7]	16[3]	22[4]	65[11]	72[12]	60[10]	90[15]	54[9]	78[13]	49[8]	84[14]
200	E_1	0.03074[1]	0.0355[6]	0.03414[4]	0.03289[3]	0.0362[8]	0.03243[2]	0.0345[5]	0.03972[11]	0.04205[12]	0.03607[7]	0.06283[15]	0.03674[9]	0.04498[13]	0.03678[10]	0.04538[14]
	E_2	0.00155[1]	0.00201[6]	0.00184[4]	0.00166[3]	0.00209[8]	0.00164[2]	0.00191[5]	0.00244[11]	0.00279[12]	0.00202[7]	0.00751[15]	0.00211[9]	0.00316[13]	0.00212[10]	0.00329[14]
	E_3	0.03416[1]	0.03944[6]	0.03793[4]	0.03655[3]	0.04022[8]	0.03603[2]	0.03833[5]	0.04414[11]	0.04672[12]	0.04008[7]	0.06982[15]	0.04082[9]	0.04997[13]	0.04086[10]	0.05042[14]
	E_4	0.01391[1]	0.01598[6]	0.0154[4]	0.01502[3]	0.01636[7]	0.0147[2]	0.01564[5]	0.01797[11]	0.01911[12]	0.01642[8]	0.0272[15]	0.01664[9]	0.02046[13]	0.01671[10]	0.02072[14]
	E_5	0.02059[1]	0.02379[6]	0.02286[4]	0.02223[3]	0.02429[7]	0.0218[2]	0.02315[5]	0.02665[11]	0.02835[12]	0.02433[8]	0.04063[15]	0.02462[9]	0.03045[13]	0.02484[10]	0.03076[14]
	E_6	0.01196[2]	0.01218[4.5]	0.01232[7]	0.01217[3]	0.01221[6]	0.01186[1]	0.01218[4.5]	0.01332[9]	0.01471[12]	0.01378[11]	0.01983[15]	0.01311[8]	0.0162[14]	0.01358[10]	0.01574[13]
	$\sum$ *Ranks*	7[1]	34.5[6]	27[4]	18[3]	44[7]	11[2]	29.5[5]	64[11]	72[12]	48[8]	90[15]	53[9]	79[13]	60[10]	83[14]
300	E_1	0.02666[2]	0.02842[5]	0.029[6]	0.02583[1]	0.02985[8]	0.02732[3]	0.02775[4]	0.03149[11]	0.03384[12]	0.03113[10]	0.05494[15]	0.03031[9]	0.03648[14]	0.02974[7]	0.03582[13]
	E_2	0.0011[2]	0.00126[5]	0.00131[6]	0.00105[1]	0.00138[7]	0.0012[4]	0.00119[3]	0.00156[11]	0.00182[12]	0.00148[9]	0.00709[15]	0.0015[10]	0.00215[14]	0.00141[8]	0.00204[13]
	E_3	0.02963[2]	0.03158[5]	0.03222[6]	0.0287[1]	0.03317[8]	0.03036[3]	0.03083[4]	0.03499[11]	0.0376[12]	0.03459[10]	0.06104[15]	0.03367[9]	0.04054[14]	0.03304[7]	0.0398[13]
	E_4	0.01206[2]	0.01284[5]	0.01313[6]	0.01178[1]	0.01347[7]	0.01235[3]	0.01257[4]	0.01427[11]	0.01536[12]	0.01416[10]	0.02362[15]	0.01371[9]	0.01656[14]	0.01353[8]	0.01632[13]
	E_5	0.01789[2]	0.01905[5]	0.01948[6]	0.01745[1]	0.02[7]	0.01833[3]	0.01864[4]	0.02116[11]	0.02278[12]	0.02102[10]	0.03526[15]	0.02034[9]	0.02463[14]	0.02009[8]	0.02423[13]
	E_6	0.00931[2]	0.00934[3]	0.00961[7]	0.00941[5]	0.00953[6]	0.00926[1]	0.00938[4]	0.01038[9]	0.01167[12]	0.011[11]	0.01679[15]	0.01027[8]	0.01252[13]	0.0104[10]	0.01253[14]
	$\sum$ *Ranks*	12[2]	28[5]	37[6]	10[1]	43[7]	17[3]	23[4]	64[11]	72[12]	60[10]	90[15]	54[9]	83[14]	48[8]	79[13]
400	E_1	0.02256[2]	0.02417[5]	0.025[6]	0.02227[1]	0.02533[8]	0.02345[3]	0.02415[4]	0.02743[11]	0.02818[12]	0.02551[9]	0.0452[15]	0.02528[7]	0.03184[13]	0.02607[10]	0.03244[14]
	E_2	0.00081[2]	0.00091[4]	0.00096[6]	0.00078[1]	0.00102[8]	0.00087[3]	0.00092[5]	0.0012[11]	0.00125[12]	0.00105[9]	0.00476[15]	0.001[7]	0.00163[13.5]	0.00106[10]	0.00163[13.5]
	E_3	0.02506[2]	0.02685[5]	0.02778[6]	0.02475[1]	0.02815[8]	0.02605[3]	0.02683[4]	0.03048[11]	0.03131[12]	0.02834[9]	0.05022[15]	0.02809[7]	0.03538[13]	0.02897[10]	0.03604[14]
	E_4	0.01022[2]	0.01095[5]	0.01129[6]	0.01012[1]	0.01145[7]	0.01062[3]	0.01091[4]	0.01242[11]	0.01281[12]	0.01155[9]	0.01958[15]	0.01147[8]	0.01452[13]	0.01184[10]	0.01474[14]
	E_5	0.01517[2]	0.01623[5]	0.01677[6]	0.01503[1]	0.017[7]	0.01575[3]	0.0162[4]	0.01843[11]	0.019[12]	0.01716[9]	0.02924[15]	0.01701[8]	0.02153[13]	0.01758[10]	0.02188[14]
	E_6	0.00786[4]	0.00781[2]	0.00805[6]	0.00782[3]	0.0082[7]	0.00775[1]	0.00788[5]	0.00847[8]	0.0099[12]	0.00915[11]	0.01333[15]	0.0086[9]	0.01041[13]	0.00872[10]	0.01067[14]
	$\sum$ *Ranks*	14[2]	26[4.5]	36[6]	8[1]	45[7]	16[3]	26[4.5]	63[11]	72[12]	56[9]	90[15]	46[8]	78.5[13]	60[10]	83.5[14]

Table 4. Numerical values for simulation measures for ($\alpha = 0.9$) under RSS.

n		MLE	ADE	CVME	MPSE	OLSE	R-ADE	WLSE	L-ADE	MNSADE	MNSALODE	AD-LSOE	KE	MNSSD	MNSSLOD	MNSLXD
20	E_1	0.06188[1]	0.06214[2]	0.06504[4]	0.0703[7]	0.06658[6]	0.06561[5]	0.06417[3]	0.07712[9]	0.09791[12]	0.0893[11]	0.1531[15]	0.07205[8]	0.11126[13]	0.08083[10]	0.11275[14]
	E_2	0.00615[2]	0.00608[1]	0.00691[4]	0.00777[7]	0.00713[6]	0.00692[5]	0.00645[3]	0.01071[10]	0.01487[12]	0.01229[11]	0.04635[15]	0.00882[8]	0.02043[13]	0.01049[9]	0.02187[14]
	E_3	0.06875[1]	0.06905[2]	0.07227[4]	0.07811[7]	0.07398[6]	0.07289[5]	0.0713[3]	0.08568[9]	0.10879[12]	0.09922[11]	0.17011[15]	0.08005[8]	0.12363[13]	0.08981[10]	0.12528[14]
	E_4	0.02744[1]	0.02808[2]	0.02902[3]	0.03301[8]	0.03005[6]	0.02956[5]	0.02913[4]	0.0335[9]	0.04458[12]	0.04164[11]	0.06145[15]	0.03199[7]	0.05144[13]	0.03729[10]	0.05157[14]
	E_5	0.04099[1]	0.04144[2]	0.04302[3]	0.04884[8]	0.04435[6]	0.04384[5]	0.04305[4]	0.04999[9]	0.06635[12]	0.06188[11]	0.09097[15]	0.04748[7]	0.07636[13]	0.05534[10]	0.07686[14]
	E_6	0.04703[2]	0.04726[3]	0.04783[4]	0.04813[6]	0.04836[7]	0.04682[1]	0.04792[5]	0.05174[9]	0.05852[12]	0.05523[11]	0.075[15]	0.05086[8]	0.06308[13]	0.05294[10]	0.06486[14]
	$\sum Ranks$	8[1]	12[2]	22[3.5]	43[7]	37[6]	26[5]	22[3.5]	55[9]	72[12]	66[11]	90[15]	46[8]	78[13]	59[10]	84[14]
60	E_1	0.03559[1]	0.03666[3]	0.03822[6]	0.04052[7]	0.03802[5]	0.03735[4]	0.03577[2]	0.04512[9]	0.05872[12]	0.05237[11]	0.09328[15]	0.04087[8]	0.06554[13]	0.04878[10]	0.06735[14]
	E_2	0.00197[1]	0.0021[3]	0.00231[6]	0.00248[7]	0.00228[5]	0.00219[4]	0.00206[2]	0.00327[9]	0.00559[12]	0.00426[11]	0.0215[15]	0.00262[8]	0.00693[13]	0.00379[10]	0.00742[14]
	E_3	0.03954[1]	0.04073[3]	0.04247[6]	0.04502[7]	0.04224[5]	0.0415[4]	0.03975[2]	0.05013[9]	0.06524[12]	0.05819[11]	0.10364[15]	0.04542[8]	0.07283[13]	0.0542[10]	0.07483[14]
	E_4	0.0161[1]	0.01652[3]	0.01744[6]	0.01865[8]	0.01719[5]	0.01692[4]	0.01622[2]	0.02019[9]	0.02688[12]	0.02394[11]	0.03784[15]	0.01837[7]	0.03006[13]	0.02228[10]	0.03092[14]
	E_5	0.02398[1]	0.0246[3]	0.02578[6]	0.0277[8]	0.02553[5]	0.02517[4]	0.0241[2]	0.02998[9]	0.04002[12]	0.03563[11]	0.05652[15]	0.02728[7]	0.04467[13]	0.03306[10]	0.04584[14]
	E_6	0.02387[3]	0.02361[2]	0.02424[7]	0.02414[5]	0.0242[6]	0.02327[1]	0.02401[4]	0.0255[9]	0.03041[12]	0.02778[11]	0.03942[15]	0.02516[8]	0.03274[13]	0.02689[10]	0.03303[14]
	$\sum Ranks$	8[1]	17[3]	37[6]	42[7]	31[5]	21[4]	14[2]	54[9]	72[12]	66[11]	90[15]	46[8]	78[13]	60[10]	84[14]
120	E_1	0.02507[1]	0.02581[2]	0.02689[6]	0.02788[7]	0.02671[5]	0.02602[3]	0.02613[4]	0.03061[9]	0.04246[12]	0.03666[11]	0.06582[15]	0.02831[8]	0.04673[13]	0.03455[10]	0.04701[14]
	E_2	0.00098[1]	0.00106[3]	0.00118[6]	0.00121[7]	0.00114[5]	0.00106[3]	0.00106[3]	0.00149[9]	0.0029[12]	0.00206[11]	0.01089[15]	0.00128[8]	0.00344[13]	0.00192[10]	0.00367[14]
	E_3	0.02786[1]	0.02868[2]	0.02988[6]	0.03098[7]	0.02968[5]	0.02891[3]	0.02904[4]	0.03401[9]	0.04718[12]	0.04073[11]	0.07313[15]	0.03146[8]	0.05193[13]	0.03838[10]	0.05224[14]
	E_4	0.01134[1]	0.01168[2]	0.01216[6]	0.01276[7.5]	0.01212[5]	0.01179[3]	0.01182[4]	0.01378[9]	0.01936[12]	0.01675[11]	0.0276[15]	0.01276[7.5]	0.02141[13]	0.01582[10]	0.02142[14]
	E_5	0.01687[1]	0.01735[2]	0.01803[6]	0.01893[7]	0.01796[5]	0.01751[3]	0.01754[4]	0.02047[9]	0.02874[12]	0.02488[11]	0.04117[15]	0.01899[8]	0.03173[13]	0.0235[10]	0.03193[14]
	E_6	0.01568[5]	0.01547[3]	0.0158[6]	0.01518[2]	0.0159[7]	0.0151[1]	0.01559[4]	0.017[9]	0.01954[12]	0.01844[11]	0.02585[15]	0.01648[8]	0.02114[13]	0.01764[10]	0.02132[14]
	$\sum Ranks$	10[1]	14[2]	36[6]	37.5[7]	32[5]	16[3]	23[4]	54[9]	72[12]	66[11]	90[15]	47.5[8]	78[13]	60[10]	84[14]
200	E_1	0.01951[1]	0.02036[3]	0.02094[6]	0.02179[8]	0.02087[4]	0.02093[5]	0.01985[2]	0.02308[9]	0.03167[12]	0.0294[11]	0.04963[15]	0.02129[7]	0.03782[14]	0.02713[10]	0.03646[13]
	E_2	$6e-04$[1]	0.00064[3]	$7e-04$[6]	0.00075[8]	0.00069[5]	0.00068[4]	0.00062[2]	0.00082[9]	0.00152[12]	0.00137[11]	0.00514[15]	0.00074[7]	0.00224[14]	0.00114[10]	0.00214[13]
	E_3	0.02168[1]	0.02262[3]	0.02327[6]	0.02421[8]	0.02319[4]	0.02326[5]	0.02205[2]	0.02565[9]	0.03519[12]	0.03267[11]	0.05514[15]	0.02366[7]	0.04202[14]	0.03014[10]	0.04051[13]
	E_4	0.00884[1]	0.00919[3]	0.00948[5.5]	0.00995[8]	0.00943[4]	0.00948[5.5]	0.00898[2]	0.01042[9]	0.01442[12]	0.01341[11]	0.02151[15]	0.00962[7]	0.01727[14]	0.01235[10]	0.01669[13]
	E_5	0.01309[1]	0.01369[3]	0.01408[5]	0.01476[8]	0.01401[4]	0.01409[6]	0.01334[2]	0.01548[9]	0.0214[12]	0.01989[11]	0.03202[15]	0.01428[7]	0.02569[14]	0.01838[10]	0.02475[13]
	E_6	0.01113[2]	0.01133[4]	0.01153[7]	0.01142[5]	0.01151[6]	0.01107[1]	0.01132[3]	0.0123[9]	0.01382[12]	0.01357[11]	0.01898[15]	0.01208[8]	0.01537[13]	0.01272[10]	0.0155[14]
	$\sum Ranks$	20[3]	18[2]	44.5[8]	43[7]	26[5]	25.5[4]	12[1]	52[9]	70[12]	64[11]	88[15]	41[6]	81[14]	58[10]	77[13]
300	E_1	0.0158[1]	0.01614[2]	0.01671[5]	0.01752[7]	0.01751[6]	0.01665[4]	0.01654[3]	0.01834[9]	0.02786[12]	0.0242[11]	0.03772[15]	0.01772[8]	0.03056[14]	0.02277[10]	0.02969[13]
	E_2	$4e-04$[1]	0.00041[2]	0.00044[5]	0.00049[7]	0.00047[6]	0.00043[3.5]	0.00043[3.5]	0.00053[9]	0.00121[12]	0.00091[11]	0.00338[15]	0.00051[8]	0.00143[14]	0.00081[10]	0.0014[13]
	E_3	0.01755[1]	0.01793[2]	0.01857[5]	0.01946[6.5]	0.01946[6.5]	0.0185[4]	0.01838[3]	0.02038[9]	0.03095[12]	0.02689[11]	0.04192[15]	0.01969[8]	0.03395[14]	0.0253[10]	0.03298[13]
	E_4	0.00716[1]	0.00732[2]	0.00756[5]	0.00798[7]	0.00793[6]	0.00754[4]	0.0075[3]	0.0083[9]	0.01264[12]	0.011[11]	0.01646[15]	0.00803[8]	0.0139[14]	0.01035[10]	0.0135[13]
	E_5	0.01062[1]	0.01086[2]	0.01124[5]	0.01183[7]	0.01178[6]	0.01119[4]	0.01113[3]	0.01231[9]	0.01875[12]	0.01635[11]	0.0245[15]	0.01191[8]	0.02067[14]	0.01538[10]	0.02006[13]
	E_6	0.00883[5]	0.00882[4]	0.00903[7]	0.00866[1]	0.00896[6]	0.00869[2]	0.00878[3]	0.00958[9]	0.01121[12]	0.01045[11]	0.01401[15]	0.00946[8]	0.01174[13]	0.00987[10]	0.01195[14]
	$\sum Ranks$	24[4]	13[1]	31[5]	34.5[6]	35.5[7]	20.5[3]	17.5[2]	53[9]	71[12]	65[11]	89[15]	47[8]	82[14]	59[10]	78[13]
400	E_1	0.0142[2]	0.01386[1]	0.01448[3]	0.01532[7]	0.0149[6]	0.01458[4]	0.01477[5]	0.01658[9]	0.02441[12]	0.02125[11]	0.03325[15]	0.01587[8]	0.02451[13]	0.01822[10]	0.02674[14]
	E_2	0.00032[2]	$3e-04$[1]	0.00033[3.5]	0.00037[7]	0.00035[6]	0.00033[3.5]	0.00034[5]	0.00043[9]	0.00095[12]	$7e-04$[11]	0.00279[15]	$4e-04$[8]	0.00096[13]	0.00054[10]	0.00115[14]
	E_3	0.01578[2]	0.0154[1]	0.01609[3]	0.01702[7]	0.01655[6]	0.0162[4]	0.01641[5]	0.01843[9]	0.02713[12]	0.02361[11]	0.03694[15]	0.01763[8]	0.02723[13]	0.02025[10]	0.02971[14]
	E_4	0.00641[2]	0.00628[1]	0.00655[3]	0.00697[7]	0.00674[6]	0.0066[4]	0.00668[5]	0.0075[9]	0.01111[12]	0.00964[11]	0.01455[15]	0.00718[8]	0.01116[13]	0.00827[10]	0.01217[14]
	E_5	0.00953[2]	0.00931[1]	0.00973[3]	0.01035[7]	0.01[6]	0.0098[4]	0.00993[5]	0.01113[9]	0.01648[12]	0.01433[11]	0.02166[15]	0.01066[8]	0.01656[13]	0.01228[10]	0.01808[14]
	E_6	0.00745[5]	0.00736[3]	0.00757[6]	0.00734[2]	0.00764[7]	0.00729[1]	0.0074[4]	0.00806[9]	0.00959[12]	0.00876[11]	0.01211[15]	0.00792[8]	0.0101[13]	0.00829[10]	0.01033[14]
	$\sum Ranks$	14[1]	20[3]	20.5[4]	36[6.5]	36[6.5]	19.5[2]	28[5]	52[8]	69[11]	70[12]	87[15]	54[9]	75[13]	58[10]	81[14]

Table 5. Numerical values for simulation measures for ($\alpha = 1.5$) under SRS.

n		MLE	ADE	CVME	MPSE	OLSE	R-ADE	WLSE	L-ADE	MNSADE	MNSALODE	AD-LSOE	KE	MNSSDE	MNSSLODE	MNSLXDE
20	E_1	0.1781[1]	0.19472[4]	0.21431[10]	0.18182[2]	0.20821[7]	0.18227[3]	0.20203[6]	0.22728[12]	0.2199[11]	0.20929[8]	0.31004[15]	0.21039[9]	0.24164[13]	0.20029[5]	0.25273[14]
	E_2	0.05411[2]	0.06502[4]	0.07603[9]	0.05159[1]	0.0752[8]	0.05602[3]	0.06828[6]	0.09182[12]	0.0801[11]	0.07056[7]	0.17482[15]	0.07617[10]	0.09406[13]	0.06622[5]	0.10497[14]
	E_3	0.11873[1]	0.12982[4]	0.14287[10]	0.12121[2]	0.1388[7]	0.12152[3]	0.13469[6]	0.15152[12]	0.1466[11]	0.13953[8]	0.20669[15]	0.14026[9]	0.1611[13]	0.13353[5]	0.16848[14]
	E_4	0.04403[1]	0.04882[4]	0.05315[9]	0.04749[3]	0.05181[7]	0.04546[2]	0.05054[5]	0.05581[11]	0.0559[12]	0.05429[10]	0.07109[15]	0.05215[8]	0.06366[13]	0.05095[6]	0.06552[14]
	E_5	0.06502[1]	0.07171[4]	0.07807[9]	0.06954[3]	0.07615[7]	0.06728[2]	0.0742[5]	0.08204[11]	0.08205[12]	0.07997[10]	0.10568[15]	0.07683[8]	0.094[13]	0.07535[6]	0.09698[14]
	E_6	0.05165[5]	0.0511[4]	0.05309[7]	0.0509[3]	0.05209[6]	0.04986[1]	0.05073[2]	0.05845[11]	0.06048[12]	0.05825[10]	0.07673[15]	0.05643[9]	0.06669[13]	0.05623[8]	0.06708[14]
	$\sum$ Ranks	11[1]	24[4]	54[10]	14[2.5]	42[7]	14[2.5]	30[5]	69[11.5]	69[11.5]	53[8.5]	90[15]	53[8.5]	78[13]	35[6]	84[14]
60	E_1	0.10411[2]	0.11113[4]	0.11889[8]	0.10388[1]	0.11608[6]	0.10682[3]	0.11399[5]	0.13124[12]	0.13101[11]	0.12327[9]	0.21699[15]	0.1237[10]	0.14304[13]	0.11792[7]	0.14859[14]
	E_2	0.01738[2]	0.01984[4]	0.02401[9]	0.01678[1]	0.0215[6]	0.01903[3]	0.02059[5]	0.02881[12]	0.02772[11]	0.02388[8]	0.09414[15]	0.02494[10]	0.03268[13]	0.0216[7]	0.036[14]
	E_3	0.06941[2]	0.07408[4]	0.07926[8]	0.06925[1]	0.07739[6]	0.07122[3]	0.076[5]	0.0875[12]	0.08734[11]	0.08218[9]	0.14466[15]	0.08246[10]	0.09536[13]	0.07861[7]	0.09906[14]
	E_4	0.02639[1]	0.02799[4]	0.02966[6]	0.0268[2]	0.02984[7]	0.02701[3]	0.02874[5]	0.03263[11]	0.03344[12]	0.03178[10]	0.05028[15]	0.03099[9]	0.03689[13]	0.03037[8]	0.03823[14]
	E_5	0.03923[1]	0.04164[4]	0.04414[7]	0.03964[2]	0.04403[6]	0.04014[3]	0.04269[5]	0.04869[11]	0.04967[12]	0.04706[10]	0.07526[15]	0.04602[9]	0.05475[13]	0.04495[8]	0.0566[14]
	E_6	0.0254[4]	0.02538[3]	0.02549[6]	0.02524[2]	0.02623[7]	0.02477[1]	0.02544[5]	0.02858[10]	0.03134[12]	0.02988[11]	0.04334[15]	0.02794[8]	0.03369[14]	0.02795[9]	0.0335[13]
	$\sum$ Ranks	12[2]	23[4]	44[7]	9[1]	38[6]	16[3]	30[5]	68[11]	69[12]	57[10]	90[15]	56[9]	79[13]	46[8]	83[14]
120	E_1	0.07152[1]	0.07848[5]	0.08313[7]	0.07329[2]	0.08329[8]	0.07365[3]	0.08116[6]	0.08947[11]	0.09257[12]	0.08473[9]	0.14931[15]	0.08638[10]	0.10195[13]	0.07605[4]	0.10511[14]
	E_2	0.0081[1]	0.00968[5]	0.01118[7]	0.00854[2]	0.01132[9]	0.00856[3]	0.01023[6]	0.01288[11]	0.01364[12]	0.0113[8]	0.04319[15]	0.01211[10]	0.01659[13]	0.00925[4]	0.01831[14]
	E_3	0.04768[1]	0.05232[5]	0.05542[7]	0.04886[2]	0.05552[8]	0.0491[3]	0.05411[6]	0.05965[11]	0.06171[12]	0.05649[9]	0.09954[15]	0.05759[10]	0.06797[13]	0.0507[4]	0.07007[14]
	E_4	0.01825[1]	0.02007[5]	0.02095[7]	0.01888[2.5]	0.02119[8]	0.01888[2.5]	0.02064[6]	0.02275[11]	0.0237[12]	0.02191[10]	0.03569[15]	0.02178[9]	0.02617[13]	0.0196[4]	0.02697[14]
	E_5	0.02708[1]	0.02975[5]	0.03117[7]	0.02803[3]	0.03143[8]	0.02792[2]	0.03065[6]	0.03377[11]	0.03516[12]	0.03246[9]	0.05342[15]	0.03249[10]	0.03892[13]	0.02911[4]	0.04007[14]
	E_6	0.01657[5]	0.01653[4]	0.01686[6]	0.01631[2]	0.01697[7]	0.01613[1]	0.01634[3]	0.01832[9]	0.02029[12]	0.0191[11]	0.02892[15]	0.0178[8]	0.02185[13]	0.01849[10]	0.02194[14]
	$\sum$ Ranks	10[1]	29[4]	41[7]	13.5[2]	48[8]	14.5[3]	33[6]	64[11]	72[12]	56[9]	90[15]	57[10]	78[13]	30[5]	84[14]
200	E_1	0.0564[3]	0.05881[4]	0.06468[8]	0.05601[2]	0.0626[5]	0.05536[1]	0.06356[6]	0.07009[11]	0.07186[12]	0.06571[9]	0.11925[15]	0.06663[10]	0.07932[14]	0.06394[7]	0.07715[13]
	E_2	0.00491[2]	0.00553[4]	0.00653[8]	0.00499[3]	0.00617[5]	0.00484[1]	0.00619[6]	0.00789[11]	0.00811[12]	0.0068[9]	0.02968[15]	0.00699[10]	0.00974[14]	0.00641[7]	0.00936[13]
	E_3	0.0376[3]	0.03921[4]	0.04312[8]	0.03734[2]	0.04174[5]	0.03691[1]	0.04238[6]	0.04673[11]	0.04791[12]	0.0438[9]	0.0795[15]	0.04442[10]	0.05288[14]	0.04263[7]	0.05143[13]
	E_4	0.01435[2.5]	0.01496[4]	0.01652[8]	0.01435[2.5]	0.01589[5]	0.01414[1]	0.01611[6]	0.01778[11]	0.0184[12]	0.01681[9]	0.0286[15]	0.01699[10]	0.02033[14]	0.01645[7]	0.01976[13]
	E_5	0.02129[2]	0.02224[4]	0.0245[8]	0.02133[3]	0.02367[5]	0.02099[1]	0.02401[6]	0.02644[11]	0.02732[12]	0.02501[9]	0.04293[15]	0.02523[10]	0.03021[14]	0.02442[7]	0.0294[13]
	E_6	0.01192[2]	0.01193[3]	0.01212[6]	0.01194[4]	0.01222[7]	0.0117[1]	0.01196[5]	0.01354[10]	0.01482[12]	0.01389[11]	0.0213[15]	0.01319[9]	0.01571[13]	0.01312[8]	0.01598[14]
	$\sum$ Ranks	14.5[2]	23[4]	46[8]	16.5[3]	32[5]	6[1]	35[6]	65[11]	72[12]	56[9]	90[15]	59[10]	83[14]	43[7]	79[13]
300	E_1	0.04631[2]	0.04883[5]	0.05253[6]	0.04473[1]	0.05264[7]	0.04713[3]	0.04839[4]	0.05808[11]	0.05931[12]	0.05473[9]	0.09622[15]	0.05482[10]	0.06245[13]	0.05278[8]	0.06652[14]
	E_2	0.00337[2]	0.00376[4.5]	0.00429[7]	0.00319[1]	0.00428[6]	0.00354[3]	0.00376[4.5]	0.00535[11]	0.00553[12]	0.00475[10]	0.02135[15]	0.00468[9]	0.00627[13]	0.00434[8]	0.00684[14]
	E_3	0.03087[2]	0.03255[5]	0.03502[6]	0.02982[1]	0.03509[7]	0.03142[3]	0.03226[4]	0.03872[11]	0.03954[12]	0.03649[9]	0.06414[15]	0.03655[10]	0.04163[13]	0.03519[8]	0.04435[14]
	E_4	0.01183[2]	0.01245[5]	0.01341[6]	0.01143[1]	0.01343[7]	0.01202[3]	0.01233[4]	0.0147[11]	0.0152[12]	0.01406[10]	0.02324[15]	0.01399[9]	0.01597[13]	0.01349[8]	0.01702[14]
	E_5	0.01758[2]	0.0185[5]	0.01992[6]	0.01699[1]	0.01995[7]	0.01787[3]	0.01833[4]	0.0219[11]	0.02258[12]	0.02083[10]	0.03476[15]	0.02078[9]	0.02374[13]	0.02005[8]	0.02532[14]
	E_6	0.00913[2]	0.00927[4]	0.00962[7]	0.00922[3]	0.00936[5]	0.00912[1]	0.00939[6]	0.01073[10]	0.01142[12]	0.01076[11]	0.01632[15]	0.01004[8]	0.01222[13]	0.01046[9]	0.0129[14]
	$\sum$ Ranks	12[2]	28.5[5]	38[6]	8[1]	39[7]	16[3]	26.5[4]	65[11]	72[12]	59[10]	90[15]	55[9]	78[13]	49[8]	84[14]
400	E_1	0.03968[1]	0.04062[2]	0.04355[5]	0.04138[3]	0.0446[7]	0.04302[4]	0.04357[6]	0.04953[11]	0.05129[12]	0.04544[8]	0.08223[15]	0.04622[9]	0.05425[13]	0.0463[10]	0.05537[14]
	E_2	0.00244[1]	0.00263[2]	0.00304[6]	0.0027[3]	0.00317[7]	0.00292[4]	0.00294[5]	0.00395[11]	0.00418[12]	0.00328[8]	0.01603[15]	0.00338[10]	0.00464[13]	0.00332[9]	0.00492[14]
	E_3	0.02646[1]	0.02708[2]	0.02903[5]	0.02758[3]	0.02974[7]	0.02868[4]	0.02905[6]	0.03302[11]	0.03419[12]	0.0303[8]	0.05482[15]	0.03081[9]	0.03617[13]	0.03087[10]	0.03691[14]
	E_4	0.01014[1]	0.01038[2]	0.0111[6]	0.01061[3]	0.01136[7]	0.01097[4]	0.01108[5]	0.01259[11]	0.01311[12]	0.0116[8]	0.02005[15]	0.01179[9]	0.0139[13]	0.01185[10]	0.01421[14]
	E_5	0.01506[1]	0.01542[2]	0.01649[5]	0.01576[3]	0.01687[7]	0.01633[4]	0.0165[6]	0.01873[11]	0.01948[12]	0.01727[8]	0.02997[15]	0.01754[9]	0.02069[13]	0.0176[10]	0.0211[14]
	E_6	0.00773[2]	0.00785[5]	0.00804[7]	0.00781[4]	0.00795[6]	0.00767[1]	0.00777[3]	0.00872[9]	0.00965[12]	0.00902[11]	0.01374[15]	0.00852[8]	0.01036[13]	0.00877[10]	0.01041[14]
	$\sum$ Ranks	7[1]	15[2]	34[6]	19[3]	41[7]	21[4]	31[5]	64[11]	72[12]	51[8]	90[15]	54[9]	78[13]	59[10]	84[14]

Table 6. Numerical values for simulation measures for ($\alpha = 1.5$) under RSS.

n		MLE	ADE	CVME	MPSE	OLSE	R-ADE	WLSE	L-ADE	MNSADE	MNSALODE	AD-LSOE	KE	MNSSDE	MNSSLODE	MNSLXDE
20	E_1	0.11164[1]	0.11178[2]	0.1179[5]	0.12226[7]	0.12017[6]	0.11378[3]	0.11506[4]	0.13294[9]	0.17401[12]	0.15779[11]	0.26818[15]	0.12237[8]	0.19721[14]	0.14671[10]	0.19296[13]
	E_2	0.01966[1]	0.02026[3]	0.0226[5]	0.02298[7]	0.02273[6]	0.01981[2]	0.02149[4]	0.03008[9]	0.04811[12]	0.03774[11]	0.14463[15]	0.02474[8]	0.06246[14]	0.03311[10]	0.06039[13]
	E_3	0.07443[1]	0.07452[2]	0.0786[5]	0.0815[7]	0.08011[6]	0.07585[3]	0.07671[4]	0.08863[9]	0.11601[12]	0.10519[11]	0.17879[15]	0.08158[8]	0.13147[14]	0.09781[10]	0.12864[13]
	E_4	0.0283[1]	0.02838[2]	0.02983[5]	0.03252[8]	0.03069[6]	0.02928[4]	0.02914[3]	0.03303[9]	0.045[12]	0.04137[11]	0.0594[15]	0.03082[7]	0.05188[14]	0.03784[10]	0.05084[13]
	E_5	0.04195[1]	0.04204[2]	0.04413[5]	0.04797[8]	0.04541[6]	0.04338[4]	0.04303[3]	0.04894[9]	0.06697[12]	0.06183[11]	0.08855[15]	0.04557[7]	0.07748[14]	0.05671[10]	0.07563[13]
	E_6	0.04831[6]	0.04695[2]	0.04864[7]	0.0483[4.5]	0.0483[4.5]	0.04691[1]	0.0475[3]	0.05287[9]	0.05783[12]	0.05409[11]	0.0747[15]	0.05186[8]	0.06377[13]	0.05301[10]	0.0643[14]
	$\sum$ *Ranks*	11[1]	13[2]	32[5]	41.5[7]	34.5[6]	17[3]	21[4]	54[9]	72[12]	66[11]	90[15]	46[8]	83[14]	60[10]	79[13]
60	E_1	0.06487[4]	0.0639[1]	0.06763[5]	0.0731[7]	0.06914[6]	0.06393[2]	0.06447[3]	0.07604[9]	0.10291[12]	0.09294[11]	0.1787[15]	0.07382[8]	0.11518[14]	0.08521[10]	0.11124[13]
	E_2	0.00659[3]	0.00647[1.5]	0.00744[5]	0.00828[7]	0.00782[6]	0.00647[1.5]	0.00666[4]	0.0093[9]	0.01731[12]	0.01316[11]	0.07321[15]	0.00877[8]	0.02209[14]	0.01181[10]	0.01949[13]
	E_3	0.04325[4]	0.0426[1]	0.04509[5]	0.04873[7]	0.04609[6]	0.04262[2]	0.04298[3]	0.05069[9]	0.06861[12]	0.06196[11]	0.11913[15]	0.04921[8]	0.07679[14]	0.05681[10]	0.07416[13]
	E_4	0.01649[4]	0.0162[1]	0.01716[5]	0.01901[8]	0.01757[6]	0.01627[2]	0.01641[3]	0.01907[9]	0.02654[12]	0.02419[11]	0.04076[15]	0.01871[7]	0.02986[14]	0.02212[10]	0.02899[13]
	E_5	0.02451[4]	0.02409[1]	0.02551[5]	0.02829[8]	0.02605[6]	0.02424[2]	0.0244[3]	0.02848[9]	0.03952[12]	0.03606[11]	0.06075[15]	0.02787[7]	0.04436[14]	0.03288[10]	0.04308[13]
	E_6	0.02396[5]	0.02386[4]	0.02462[7]	0.02357[2]	0.02402[6]	0.02309[1]	0.02366[3]	0.02573[8]	0.02978[12]	0.02783[11]	0.04207[15]	0.02577[9]	0.03152[14]	0.02704[10]	0.03146[13]
	$\sum$ *Ranks*	24[4]	9.5[1]	32[5]	39[7]	36[6]	10.5[2]	19[3]	53[9]	72[12]	66[11]	90[15]	47[8]	84[14]	60[10]	78[13]
120	E_1	0.04276[1]	0.04543[3]	0.04789[5]	0.0511[8]	0.048[6]	0.04667[4]	0.04387[2]	0.05337[9]	0.07549[12]	0.06511[11]	0.12143[15]	0.04999[7]	0.08294[14]	0.06136[10]	0.08197[13]
	E_2	0.00286[1]	0.00318[3]	0.00367[5]	0.004[8]	0.0037[6]	0.00332[4]	0.00302[2]	0.00459[9]	0.00898[12]	0.00661[11]	0.03793[15]	0.00397[7]	0.01054[14]	0.00611[10]	0.01053[13]
	E_3	0.0285[1]	0.03029[3]	0.03193[5]	0.03407[8]	0.032[6]	0.03111[4]	0.02925[2]	0.03558[9]	0.05033[12]	0.04341[11]	0.08095[15]	0.03332[7]	0.05529[14]	0.04091[10]	0.05465[13]
	E_4	0.01091[1]	0.0116[3]	0.01217[5]	0.01312[8]	0.01221[6]	0.01194[4]	0.01118[2]	0.01357[9]	0.01939[12]	0.01679[11]	0.02842[15]	0.01267[7]	0.02147[14]	0.01577[10]	0.02121[13]
	E_5	0.01622[1]	0.01722[3]	0.01813[5]	0.01956[8]	0.01818[6]	0.01772[4]	0.01663[2]	0.02015[9]	0.02887[12]	0.02494[11]	0.04244[15]	0.01892[7]	0.03195[14]	0.02344[10]	0.03156[13]
	E_6	0.01535[4]	0.0152[2]	0.01538[5]	0.01549[6]	0.01581[7]	0.01503[1]	0.01523[3]	0.017[9]	0.01943[12]	0.01808[11]	0.02664[15]	0.01666[8]	0.02092[14]	0.01755[10]	0.02089[13]
	$\sum$ *Ranks*	9[1]	17[3]	30[5]	46[8]	37[6]	21[4]	13[2]	54[9]	72[12]	66[11]	90[15]	43[7]	84[14]	60[10]	78[13]
200	E_1	0.03526[2]	0.03545[3]	0.03653[5]	0.03794[7]	0.03667[6]	0.03607[4]	0.0348[1]	0.04266[9]	0.05944[12]	0.05061[11]	0.09225[15]	0.03993[8]	0.06488[14]	0.04809[10]	0.06293[13]
	E_2	0.00197[2]	0.00198[3]	0.00206[5]	0.00221[7]	0.00211[6]	0.00203[4]	0.00194[1]	0.00276[9]	0.00556[12]	0.00399[11]	0.02471[15]	0.00249[8]	0.00651[14]	0.00376[10]	0.00626[13]
	E_3	0.02351[2]	0.02363[3]	0.02435[5]	0.02529[7]	0.02444[6]	0.02405[4]	0.0232[1]	0.02844[9]	0.03962[12]	0.03374[11]	0.0615[15]	0.02662[8]	0.04325[14]	0.03206[10]	0.04195[13]
	E_4	0.00901[2]	0.00906[3]	0.00933[5]	0.00977[7]	0.00934[6]	0.00922[4]	0.00884[1]	0.01087[9]	0.01525[12]	0.01295[11]	0.02189[15]	0.01016[8]	0.01668[14]	0.01236[10]	0.01614[13]
	E_5	0.01337[2]	0.01346[3]	0.01387[5]	0.01452[7]	0.01388[6]	0.01371[4]	0.01317[1]	0.01616[9]	0.02268[12]	0.01931[11]	0.03266[15]	0.0151[8]	0.02483[14]	0.01836[10]	0.02399[13]
	E_6	0.01114[2]	0.0112[4]	0.01129[6]	0.01119[3]	0.01139[7]	0.01113[1]	0.01126[5]	0.0122[9]	0.01419[12]	0.01335[11]	0.01934[15]	0.01206[8]	0.01535[13]	0.01256[10]	0.01576[14]
	$\sum$ *Ranks*	12[2]	19[3]	31[5]	38[7]	37[6]	21[4]	10[1]	54[9]	72[12]	66[11]	90[15]	48[8]	83[14]	60[10]	79[13]
300	E_1	0.02801[1]	0.0291[3]	0.02999[6]	0.03157[7]	0.02903[2]	0.02945[4]	0.0298[5]	0.03317[9]	0.04666[12]	0.04301[11]	0.07089[15]	0.0323[8]	0.05398[13]	0.0401[10]	0.05409[14]
	E_2	0.00122[1]	0.00134[3]	0.00138[4.5]	0.00157[7]	0.00133[2]	0.0014[6]	0.00138[4.5]	0.00171[9]	0.00351[12]	0.00289[11]	0.01232[15]	0.00165[8]	0.00457[13]	0.00245[10]	0.00458[14]
	E_3	0.01867[1]	0.0194[3]	0.01999[6]	0.02105[7]	0.01935[2]	0.01963[4]	0.01986[5]	0.02212[9]	0.03111[12]	0.02867[11]	0.04726[15]	0.02153[8]	0.03599[13]	0.02673[10]	0.03606[14]
	E_4	0.00713[1]	0.00743[3]	0.00766[6]	0.00812[7]	0.00741[2]	0.00751[4]	0.00762[5]	0.00845[9]	0.01195[12]	0.01105[11]	0.01732[15]	0.00824[8]	0.01385[13]	0.01026[10]	0.01391[14]
	E_5	0.01061[1]	0.01104[3]	0.01139[6]	0.01206[7]	0.01101[2]	0.01119[4]	0.0113[5]	0.01256[9]	0.01776[12]	0.01637[11]	0.02581[15]	0.01225[8]	0.0206[13]	0.01527[10]	0.02069[14]
	E_6	0.0086[2.5]	0.00859[1]	0.00887[6]	0.0086[2.5]	0.0089[7]	0.00864[4]	0.00874[5]	0.00937[8]	0.01108[12]	0.01022[11]	0.01485[15]	0.00941[9]	0.01206[13]	0.01005[10]	0.01234[14]
	$\sum$ *Ranks*	7.5[1]	16[2]	34.5[6]	37.5[7]	17[3]	26[4]	29.5[5]	53[9]	72[12]	66[11]	90[15]	49[8]	78[13]	60[10]	84[14]
400	E_1	0.02399[1]	0.02468[3]	0.02674[7]	0.02666[6]	0.02465[2]	0.02524[4]	0.02538[5]	0.02889[9]	0.04024[12]	0.03538[11]	0.06564[15]	0.0282[8]	0.04484[13]	0.03358[10]	0.04656[14]
	E_2	0.00091[1]	0.00095[2.5]	0.00109[6]	0.00111[7]	0.00095[2.5]	0.00101[4]	0.00102[5]	0.00133[9]	0.00253[12]	0.002[11]	0.01187[15]	0.00122[8]	0.00321[13]	0.00179[10]	0.00337[14]
	E_3	0.016[1]	0.01645[3]	0.01783[7]	0.01777[6]	0.01643[2]	0.01683[4]	0.01692[5]	0.01926[9]	0.02682[12]	0.02359[11]	0.04376[15]	0.0188[8]	0.02989[13]	0.02239[10]	0.03104[14]
	E_4	0.00611[1]	0.0063[3]	0.00682[6]	0.00683[7]	0.00629[2]	0.00644[4]	0.00646[5]	0.00736[9]	0.01031[12]	0.00905[11]	0.01597[15]	0.00719[8]	0.01149[13]	0.00863[10]	0.01196[14]
	E_5	0.00908[1]	0.00936[3]	0.01014[6]	0.01016[7]	0.00935[2]	0.00959[4]	0.00962[5]	0.01094[9]	0.01533[12]	0.01348[11]	0.02378[15]	0.01069[8]	0.0171[13]	0.01282[10]	0.01779[14]
	E_6	0.00722[2]	0.0074[4]	0.00744[6]	0.00743[5]	0.00748[7]	0.00708[1]	0.00731[3]	0.00804[9]	0.00924[12]	0.00848[11]	0.0129[15]	0.00793[8]	0.00995[13]	0.0084[10]	0.00998[14]
	$\sum$ *Ranks*	7[1]	18.5[3]	38[6.5]	38[6.5]	17.5[2]	21[4]	28[5]	54[9]	72[12]	66[11]	90[15]	48[8]	78[13]	60[10]	84[14]

Table 7. Numerical values for simulation measures for ($\alpha = 2.5$) under SRS.

n		MLE	ADE	CVME	MPSE	OLSE	R-ADE	WLSE	L-ADE	MNSADE	MNSALODE	AD-LSOE	KE	MSSD	MNSSLODE	MNSLXDE
20	E_1	0.34463[3]	0.34966[6]	0.34859[4]	0.33061[1]	0.36932[10]	0.33394[2]	0.3558[7]	0.41532[12]	0.36833[9]	0.36233[8]	0.55763[15]	0.38147[11]	0.43792[14]	0.34871[5]	0.41887[13]
	E_2	0.2066[4]	0.20797[5]	0.22038[8]	0.17637[1]	0.24091[10]	0.19109[2]	0.2158[7]	0.29693[13]	0.22908[9]	0.21066[6]	0.55481[15]	0.25085[11]	0.3156[14]	0.20154[3]	0.28538[12]
	E_3	0.13785[3]	0.13986[6]	0.13944[4]	0.13224[1]	0.14773[10]	0.13358[2]	0.14232[7]	0.16613[12]	0.14733[9]	0.14493[8]	0.22305[15]	0.15259[11]	0.17517[14]	0.13949[5]	0.16755[13]
	E_4	0.04653[2]	0.0479[4]	0.04863[5]	0.04787[3]	0.05081[8]	0.04621[1]	0.04874[6]	0.0562[12]	0.05256[11]	0.05237[10]	0.0721[15]	0.05196[9]	0.0629[14]	0.04912[7]	0.06193[13]
	E_5	0.06776[1]	0.07001[3]	0.07108[5]	0.07016[4]	0.07445[8]	0.0679[2]	0.07171[6]	0.08283[12]	0.07695[11]	0.07598[9]	0.10568[15]	0.07645[10]	0.09289[14]	0.0721[7]	0.09135[13]
	E_6	0.0509[3]	0.05131[6]	0.05326[7]	0.04936[1]	0.05108[4]	0.04938[2]	0.0513[5]	0.05839[11]	0.05989[12]	0.05766[10]	0.08017[15]	0.05652[9]	0.06785[14]	0.0547[8]	0.06711[13]
	$\sum$ *Ranks*	16[3]	30[4]	33[5]	11[1.5]	50[8]	11[1.5]	38[7]	72[12]	61[10.5]	51[9]	90[15]	61[10.5]	84[14]	35[6]	77[13]
60	E_1	0.18441[2]	0.20438[6]	0.214[9]	0.18768[3]	0.20558[7]	0.1839[1]	0.1967[4]	0.23128[12]	0.22684[11]	0.21335[8]	0.38622[15]	0.22377[10]	0.25352[14]	0.19785[5]	0.25212[13]
	E_2	0.05564[3]	0.06651[6]	0.07416[9]	0.0553[2]	0.07026[7]	0.05381[1]	0.0619[4]	0.08956[12]	0.0889[11]	0.07115[8]	0.30326[15]	0.0802[10]	0.10506[13]	0.06449[5]	0.10968[14]
	E_3	0.07376[2]	0.08175[6]	0.0856[9]	0.07507[3]	0.08223[7]	0.07356[1]	0.07868[4]	0.09251[12]	0.09074[11]	0.08534[8]	0.15449[15]	0.08951[10]	0.10141[14]	0.07914[5]	0.10085[13]
	E_4	0.02598[2]	0.02869[6]	0.03008[8]	0.02701[3]	0.02907[7]	0.02596[1]	0.02782[4]	0.0322[12]	0.03183[11]	0.03065[9]	0.0489[15]	0.03104[10]	0.03635[14]	0.02819[5]	0.03605[13]
	E_5	0.03824[1]	0.04237[6]	0.04444[8]	0.03989[3]	0.04284[7]	0.0383[2]	0.04099[4]	0.04756[12]	0.04709[11]	0.04518[9]	0.07312[15]	0.04598[10]	0.05379[14]	0.04146[5]	0.05335[13]
	E_6	0.02512[3]	0.02524[5]	0.02584[6]	0.02459[2]	0.02611[7]	0.02409[1]	0.02521[4]	0.02799[9]	0.03041[12]	0.02829[11]	0.04405[15]	0.028[10]	0.03321[13]	0.02764[8]	0.03349[14]
	$\sum$ *Ranks*	13[2]	35[6]	49[8]	16[3]	42[7]	7[1]	24[4]	69[12]	67[11]	53[9]	90[15]	60[10]	82[14]	33[5]	80[13]
120	E_1	0.12775[1]	0.1335[4]	0.14757[8]	0.1278[2]	0.15531[10]	0.13204[3]	0.14063[5]	0.16349[12]	0.16111[11]	0.14316[6]	0.27337[15]	0.15529[9]	0.18604[14]	0.14695[7]	0.17806[13]
	E_2	0.02723[2]	0.028[4]	0.03473[8]	0.02532[1]	0.03893[10]	0.02756[3]	0.0311[5]	0.04204[12]	0.04159[11]	0.03225[6]	0.15124[15]	0.03832[9]	0.05508[14]	0.03351[7]	0.05068[13]
	E_3	0.0511[1]	0.0534[4]	0.05903[8]	0.05112[2]	0.06213[10]	0.05282[3]	0.05625[5]	0.0654[12]	0.06444[11]	0.05726[6]	0.10935[15]	0.06212[9]	0.07441[14]	0.05878[7]	0.07123[13]
	E_4	0.01799[1]	0.01906[4]	0.02084[7]	0.01843[2]	0.02198[9]	0.0188[3]	0.01992[5]	0.02306[12]	0.02301[11]	0.02055[6]	0.03568[15]	0.02202[10]	0.02678[14]	0.02107[8]	0.02543[13]
	E_5	0.02658[1]	0.02803[4]	0.03075[7]	0.02717[2]	0.03248[9]	0.02774[3]	0.02938[5]	0.03409[12]	0.03392[11]	0.0303[6]	0.05327[15]	0.0325[10]	0.03967[14]	0.03107[8]	0.03767[13]
	E_6	0.01604[2]	0.01607[3]	0.01701[6]	0.01621[4]	0.01704[7]	0.01565[1]	0.01645[5]	0.01865[10]	0.01969[12]	0.01875[11]	0.02851[15]	0.01831[9]	0.02187[14]	0.01773[8]	0.02141[13]
	$\sum$ *Ranks*	8[1]	23[4]	44[7]	13[2]	55[9]	16[3]	30[5]	70[12]	67[11]	41[6]	90[15]	56[10]	84[14]	45[8]	78[13]
200	E_1	0.1043[2]	0.10922[4]	0.11762[10]	0.10172[1]	0.11511[8]	0.10597[3]	0.11004[5]	0.12282[11]	0.12656[12]	0.11278[7]	0.21329[15]	0.11729[9]	0.13776[13]	0.11093[6]	0.14076[14]
	E_2	0.01711[2]	0.01881[4]	0.02205[10]	0.0162[1]	0.02084[8]	0.0174[3]	0.01941[6]	0.02427[11]	0.02489[12]	0.02028[7]	0.09586[15]	0.02135[9]	0.02976[13]	0.01929[5]	0.03189[14]
	E_3	0.04172[2]	0.04369[4]	0.04705[10]	0.04069[1]	0.04604[8]	0.04239[3]	0.04402[5]	0.04913[11]	0.05063[12]	0.04511[7]	0.08532[15]	0.04692[9]	0.05511[13]	0.04437[6]	0.05631[14]
	E_4	0.01475[2]	0.01543[4]	0.01658[9]	0.01456[1]	0.01634[8]	0.01494[3]	0.01558[5]	0.01737[11]	0.01799[12]	0.0161[7]	0.02855[15]	0.01668[10]	0.01983[13]	0.01585[6]	0.02003[14]
	E_5	0.02186[2]	0.02282[4]	0.02454[9]	0.02153[1]	0.02415[8]	0.02214[3]	0.02305[5]	0.02567[11]	0.0266[12]	0.02379[7]	0.04237[15]	0.02462[10]	0.02933[13]	0.02342[6]	0.02964[14]
	E_6	0.01176[3]	0.01205[5]	0.01239[7]	0.01153[2]	0.01227[6]	0.01127[1]	0.01182[4]	0.01347[11]	0.01447[12]	0.01343[10]	0.02091[15]	0.01301[8]	0.01552[13]	0.01308[9]	0.01585[14]
	$\sum$ *Ranks*	13[2]	25[4]	55[9.5]	7[1]	46[8]	16[3]	30[5]	66[11]	72[12]	45[7]	90[15]	55[9.5]	78[13]	38[6]	84[14]
300	E_1	0.08122[1]	0.09158[5]	0.09847[10]	0.08203[2]	0.09315[7]	0.08638[3]	0.08887[4]	0.10191[11]	0.10564[12]	0.0958[8]	0.16571[15]	0.09597[9]	0.11692[14]	0.09182[6]	0.1153[13]
	E_2	0.01031[1]	0.01299[5]	0.01508[10]	0.01056[2]	0.0137[7]	0.0118[3]	0.01217[4]	0.01645[11]	0.01809[12]	0.01448[8]	0.05118[15]	0.01451[9]	0.02154[14]	0.01317[6]	0.02104[13]
	E_3	0.03249[1]	0.03663[5]	0.03939[10]	0.03281[2]	0.03726[7]	0.03455[3]	0.03555[4]	0.04076[11]	0.04226[12]	0.03832[8]	0.06628[15]	0.03839[9]	0.04677[14]	0.03673[6]	0.04612[13]
	E_4	0.0115[1]	0.01299[5]	0.01388[10]	0.01173[2]	0.01316[7]	0.0122[3]	0.01259[4]	0.01446[11]	0.01505[12]	0.01371[9]	0.02272[15]	0.01353[8]	0.01671[14]	0.01308[6]	0.01643[13]
	E_5	0.01701[1]	0.01921[5]	0.0206[10]	0.01733[2]	0.0195[7]	0.01804[3]	0.01863[4]	0.02135[11]	0.0223[12]	0.02023[9]	0.0337[15]	0.02004[8]	0.02473[14]	0.01933[6]	0.02432[13]
	E_6	0.00908[2]	0.0091[3]	0.00954[7]	0.00918[5]	0.00942[6]	0.00891[1]	0.00913[4]	0.0106[10.5]	0.01125[12]	0.0106[10.5]	0.01635[15]	0.01003[8]	0.01226[14]	0.0102[9]	0.01206[13]
	$\sum$ *Ranks*	7[1]	28[5]	57[10]	15[2]	41[7]	16[3]	24[4]	65.5[11]	72[12]	52.5[9]	90[15]	51[8]	84[14]	39[6]	78[13]
400	E_1	0.07128[2]	0.07664[4]	0.08142[8]	0.06884[1]	0.08107[7]	0.07763[5]	0.07584[3]	0.08463[11]	0.08901[12]	0.08349[10]	0.14237[15]	0.08281[9]	0.0981[13]	0.07904[6]	0.09946[14]
	E_2	0.00792[2]	0.00956[5]	0.01061[8]	0.00747[1]	0.01033[7]	0.00922[4]	0.00919[3]	0.01144[11]	0.01263[12]	0.01103[9]	0.04174[15]	0.01122[10]	0.01524[13]	0.00989[6]	0.01562[14]
	E_3	0.02851[2]	0.03066[4]	0.03257[8]	0.02754[1]	0.03243[7]	0.03105[5]	0.03034[3]	0.03385[11]	0.0356[12]	0.0334[10]	0.05695[15]	0.03312[9]	0.03924[13]	0.03162[6]	0.03978[14]
	E_4	0.01012[2]	0.01085[4]	0.01151[7]	0.0098[1]	0.01152[8]	0.011[5]	0.01073[3]	0.012[11]	0.01265[12]	0.0119[10]	0.01946[15]	0.01172[9]	0.01401[13]	0.01126[6]	0.0142[14]
	E_5	0.01497[2]	0.01606[4]	0.01704[8]	0.0145[1]	0.01701[7]	0.01625[5]	0.01589[3]	0.01776[11]	0.01871[12]	0.0176[10]	0.02889[15]	0.01734[9]	0.02072[13]	0.01665[6]	0.02101[14]
	E_6	0.0076[2]	0.00762[3]	0.00793[7]	0.00778[4]	0.0079[6]	0.00751[1]	0.00786[5]	0.0089[11]	0.00946[12]	0.00878[10]	0.01367[15]	0.00846[8]	0.01043[14]	0.00856[9]	0.01029[13]
	$\sum$ *Ranks*	12[2]	24[4]	46[8]	9[1]	42[7]	25[5]	20[3]	66[11]	72[12]	59[10]	90[15]	54[9]	79[13]	39[6]	83[14]

Table 8. Numerical values for simulation measures for (α = 2.5) under RSS.

n		MLE	ADE	CVME	MPSE	OLSE	R-ADE	WLSE	L-ADE	MNSADE	MNSALODE	AD-LSOE	KE	MNSSD	MNSSLODE	MNSLXDE
20	E_1	0.19632[2]	0.19598[1]	0.2146[4]	0.22806[7]	0.21976[6]	0.19683[3]	0.21709[5]	0.2464[9]	0.30535[12]	0.28592[11]	0.45815[15]	0.23013[8]	0.34253[14]	0.25925[10]	0.34012[13]
	E_2	0.06229[2]	0.06138[1]	0.07461[4]	0.08206[7]	0.0787[6]	0.06308[3]	0.07586[5]	0.10385[9]	0.15028[12]	0.12346[11]	0.42268[15]	0.08546[8]	0.19182[14]	0.10506[10]	0.18956[13]
	E_3	0.07853[2]	0.07839[1]	0.08584[4]	0.09122[7]	0.0879[6]	0.07873[3]	0.08684[5]	0.09856[9]	0.12214[12]	0.11437[11]	0.18326[15]	0.09205[8]	0.13701[14]	0.1037[10]	0.13605[13]
	E_4	0.02742[1]	0.02776[2]	0.03006[4]	0.03369[8]	0.03079[6]	0.02796[3]	0.03066[5]	0.03382[9]	0.04339[12]	0.04262[11]	0.05703[15]	0.03206[7]	0.04964[13]	0.03735[10]	0.04972[14]
	E_5	0.04046[1]	0.04084[2]	0.04415[4]	0.04984[8]	0.04555[6]	0.04145[3]	0.04543[5]	0.05[9]	0.06411[12]	0.06253[11]	0.08427[15]	0.04752[7]	0.07369[14]	0.05523[10]	0.07345[13]
	E_6	0.04766[5]	0.04674[2]	0.0486[7]	0.04726[4]	0.04849[6]	0.04629[1]	0.04725[3]	0.0534[10]	0.05722[12]	0.05517[11]	0.07493[15]	0.05153[8]	0.06391[13]	0.05181[9]	0.06405[14]
	$\sum$ *Ranks*	13[2]	9[1]	27[4]	41[7]	36[6]	16[3]	28[5]	55[9]	72[12]	66[11]	90[15]	46[8]	82[14]	59[10]	80[13]
60	E_1	0.11225[2]	0.10909[1]	0.11964[5]	0.12779[7]	0.12475[6]	0.11521[4]	0.11355[3]	0.13488[9]	0.18865[12]	0.16874[11]	0.30297[15]	0.13309[8]	0.20519[13]	0.15231[10]	0.2099[14]
	E_2	0.0203[3]	0.01979[1]	0.02272[5]	0.02522[7]	0.02419[6]	0.02115[4]	0.02022[2]	0.02918[9]	0.05666[12]	0.04382[11]	0.22513[15]	0.02737[8]	0.06678[13]	0.03741[10]	0.07039[14]
	E_3	0.0449[2]	0.04363[1]	0.04786[5]	0.05111[7]	0.0499[6]	0.04608[4]	0.04542[3]	0.05395[9]	0.07546[12]	0.0675[11]	0.12119[15]	0.05323[8]	0.08208[13]	0.06092[10]	0.08396[14]
	E_4	0.01583[2]	0.01537[1]	0.01695[5]	0.01852[7]	0.01776[6]	0.01638[4]	0.0162[3]	0.01906[9]	0.02702[12]	0.02445[11]	0.03818[15]	0.01875[8]	0.02952[13]	0.02206[10]	0.03031[14]
	E_5	0.0234[2]	0.02278[1]	0.02511[5]	0.02745[7]	0.0262[6]	0.02423[4]	0.02386[3]	0.02816[9]	0.04009[12]	0.03613[11]	0.05678[15]	0.02774[8]	0.04386[13]	0.03259[10]	0.04488[14]
	E_6	0.02351[4]	0.0237[5]	0.0242[7]	0.02291[2]	0.02397[6]	0.02287[1]	0.02334[3]	0.02603[9]	0.02905[12]	0.02789[11]	0.04019[15]	0.02575[8]	0.03185[13]	0.02659[10]	0.0324[14]
	$\sum$ *Ranks*	15[2]	10[1]	32[5]	37[7]	36[6]	21[4]	17[3]	54[9]	72[12]	66[11]	90[15]	48[8]	78[13]	60[10]	84[14]
120	E_1	0.08123[2]	0.08223[3]	0.08494[4]	0.0932[8]	0.08514[5]	0.08731[6.5]	0.07997[1]	0.09458[9]	0.13657[12]	0.1221[11]	0.22068[15]	0.08731[6.5]	0.14778[13]	0.10825[10]	0.14894[14]
	E_2	0.01088[3]	0.01084[2]	0.01115[4]	0.01359[8]	0.0113[5]	0.01183[6]	0.00987[1]	0.0144[9]	0.02999[12]	0.02297[11]	0.117[15]	0.01216[7]	0.03547[14]	0.01839[10]	0.03431[13]
	E_3	0.03249[2]	0.03289[3]	0.03398[4]	0.03728[8]	0.03405[5]	0.03492[6.5]	0.03199[1]	0.03783[9]	0.05463[12]	0.04884[11]	0.08827[15]	0.03492[6.5]	0.05911[13]	0.0433[10]	0.05958[14]
	E_4	0.01152[2]	0.01167[3]	0.01204[4]	0.01339[8.5]	0.01211[5]	0.01242[7]	0.01135[1]	0.01339[8.5]	0.0195[12]	0.01758[11]	0.02869[15]	0.0124[6]	0.02119[13]	0.01555[10]	0.02157[14]
	E_5	0.01703[2]	0.01723[3]	0.01781[4]	0.01982[9]	0.01789[5]	0.01834[7]	0.01678[1]	0.0198[8]	0.02885[12]	0.02601[11]	0.04266[15]	0.01833[6]	0.03137[13]	0.02298[10]	0.03193[14]
	E_6	0.01504[2]	0.01514[4]	0.01537[7]	0.01509[3]	0.01535[6]	0.01502[1]	0.01515[5]	0.01705[9]	0.01922[12]	0.01769[11]	0.02715[15]	0.01664[8]	0.02102[14]	0.01713[10]	0.02091[13]
	$\sum$ *Ranks*	13[2]	18[3]	27[4]	44.5[8]	31[5]	34[6]	10[1]	52.5[9]	72[12]	66[11]	90[15]	40[7]	80[13]	60[10]	82[14]
200	E_1	0.06287[3]	0.06213[1]	0.06958[7]	0.06864[6]	0.06615[5]	0.06324[4]	0.06279[2]	0.07408[9]	0.10254[12]	0.09085[11]	0.16683[15]	0.07267[8]	0.11805[14]	0.08717[10]	0.11608[13]
	E_2	0.00625[3]	0.00615[1]	0.00753[7]	0.00727[6]	0.00691[5]	0.00629[4]	0.00624[2]	0.00862[9]	0.0167[12]	0.01278[11]	0.07593[15]	0.00838[8]	0.02202[14]	0.01191[10]	0.02132[13]
	E_3	0.02515[3]	0.02485[1]	0.02783[7]	0.02745[6]	0.02646[5]	0.02529[4]	0.02512[2]	0.02963[9]	0.04101[12]	0.03634[11]	0.06673[15]	0.02907[8]	0.04722[14]	0.03487[10]	0.04643[13]
	E_4	0.00891[3]	0.0088[1]	0.00985[7]	0.00981[6]	0.00941[5]	0.00894[4]	0.0089[2]	0.01051[9]	0.01466[12]	0.01299[11]	0.02209[15]	0.01031[8]	0.01699[14]	0.01246[10]	0.01665[13]
	E_5	0.0132[3]	0.01302[1]	0.01458[7]	0.01452[6]	0.01389[5]	0.01326[4]	0.01316[2]	0.01554[9]	0.02169[12]	0.01925[11]	0.03271[15]	0.01523[8]	0.02515[14]	0.01843[10]	0.02461[13]
	E_6	0.01101[4]	0.01094[3]	0.01145[7]	0.01087[2]	0.01133[6]	0.0107[1]	0.01111[5]	0.01231[9]	0.01377[12]	0.01308[11]	0.01985[15]	0.01211[8]	0.01525[14]	0.01246[10]	0.01504[13]
	$\sum$ *Ranks*	19[3]	8[1]	42[7]	32[6]	31[5]	21[4]	15[2]	54[9]	72[12]	66[11]	90[15]	48[8]	84[14]	60[10]	78[13]
300	E_1	0.0512[1]	0.05262[3]	0.05329[5]	0.05509[7]	0.05517[8]	0.05189[2]	0.05266[4]	0.05836[9]	0.08433[12]	0.07474[11]	0.13028[15]	0.05424[6]	0.09947[14]	0.06905[10]	0.09052[13]
	E_2	0.00414[1]	0.00423[2]	0.0045[5]	0.0047[6]	0.00497[8]	0.00424[3]	0.00437[4]	0.00538[9]	0.01101[12]	0.00871[11]	0.04943[15]	0.00479[7]	0.01527[14]	0.00744[10]	0.01363[13]
	E_3	0.02048[1]	0.02105[3]	0.02132[5]	0.02204[7]	0.02207[8]	0.02076[2]	0.02107[4]	0.02334[9]	0.03373[12]	0.02989[11]	0.05211[15]	0.0217[6]	0.03979[14]	0.02762[10]	0.03621[13]
	E_4	0.00727[1]	0.00746[3]	0.00756[5]	0.00789[8]	0.00781[7]	0.00737[2]	0.00747[4]	0.0083[9]	0.01203[12]	0.01068[11]	0.01741[15]	0.00767[6]	0.01426[14]	0.00986[10]	0.01297[13]
	E_5	0.01074[1]	0.01104[3]	0.0112[5]	0.01166[8]	0.01156[7]	0.01091[2]	0.01107[4]	0.01228[9]	0.01782[12]	0.01578[11]	0.02583[15]	0.01136[6]	0.0211[14]	0.01459[10]	0.0192[13]
	E_6	0.0086[4]	0.00879[5]	0.00889[7]	0.00842[2]	0.00883[6]	0.0084[1]	0.00851[3]	0.00945[8]	0.01089[12]	0.01014[11]	0.01492[15]	0.00953[9]	0.0121[14]	0.00987[10]	0.01163[13]
	$\sum$ *Ranks*	9[1]	19[3]	32[5]	38[6]	44[8]	12[2]	23[4]	53[9]	72[12]	66[11]	90[15]	40[7]	84[14]	60[10]	78[13]
400	E_1	0.04214[1]	0.0439[2]	0.04609[4]	0.0472[7]	0.04622[5]	0.04628[6]	0.04555[3]	0.0539[9]	0.07311[12]	0.06385[11]	0.11265[15]	0.04979[8]	0.08235[14]	0.05939[10]	0.0792[13]
	E_2	0.00287[1]	0.00313[2]	0.00337[6]	0.00354[7]	0.00333[4]	0.00335[5]	0.00316[3]	0.00453[9]	0.00836[12]	0.00637[11]	0.03578[15]	0.00392[8]	0.01092[14]	0.00576[10]	0.01007[13]
	E_3	0.01686[1]	0.01756[2]	0.01844[4]	0.01888[7]	0.01849[5]	0.01851[6]	0.01822[3]	0.02156[9]	0.02925[12]	0.02554[11]	0.04506[15]	0.01991[8]	0.03294[14]	0.02376[10]	0.03168[13]
	E_4	0.00598[1]	0.00622[2]	0.00655[4]	0.00673[7]	0.00657[5]	0.00658[6]	0.00645[3]	0.00764[9]	0.0104[12]	0.0091[11]	0.01521[15]	0.00706[8]	0.01172[14]	0.00848[10]	0.01127[13]
	E_5	0.00885[1]	0.00922[2]	0.00968[4]	0.00996[7]	0.00971[5]	0.00973[6]	0.00955[3]	0.01131[9]	0.0154[12]	0.01346[11]	0.02253[15]	0.01045[8]	0.01736[14]	0.01256[10]	0.01668[13]
	E_6	0.00717[4]	0.00719[5]	0.00739[7]	0.00709[2]	0.00734[6]	0.00701[1]	0.00715[3]	0.00812[9]	0.00912[12]	0.00856[11]	0.01256[15]	0.00772[8]	0.00993[14]	0.00824[10]	0.00983[13]
	$\sum$ *Ranks*	9[1]	15[2]	29[4]	37[7]	30[5.5]	30[5.5]	18[3]	54[9]	72[12]	66[11]	90[15]	48[8]	84[14]	60[10]	78[13]

Table 9. Numerical values for simulation measures for (α = 4.0) under SRS.

n		MLE	ADE	CVME	MPSE	OLSE	R-ADE	WLSE	L-ADE	MNSADE	MNSALODE	AD-LSOE	KE	MNSSDE	MNSSLODE	MNSLXDE
20	E_1	0.57099[3]	0.60618[6]	0.64923[9]	0.56685[2]	0.63868[8]	0.55659[1]	0.61823[7]	0.72898[13]	0.65329[10]	0.59376[4]	0.94134[15]	0.65905[11]	0.73751[14]	0.60543[5]	0.70707[12]
	E_2	0.55038[3]	0.63489[6]	0.70744[8]	0.51499[1]	0.71722[9]	0.51555[2]	0.6644[7]	0.89834[14]	0.72095[10]	0.57954[4]	1.54463[15]	0.73076[11]	0.89006[13]	0.60361[5]	0.83755[12]
	E_3	0.14275[3]	0.15154[6]	0.16231[9]	0.14171[2]	0.15967[8]	0.13915[1]	0.15456[7]	0.18225[13]	0.16332[10]	0.14844[4]	0.23534[15]	0.16476[11]	0.18438[14]	0.15136[5]	0.17677[12]
	E_4	0.04586[2]	0.0479[4]	0.05153[8]	0.04762[3]	0.05183[9]	0.0457[1]	0.04905[5]	0.05763[12]	0.05398[11]	0.0506[7]	0.07048[15]	0.05325[10]	0.06221[14]	0.04952[6]	0.05886[13]
	E_5	0.06604[1]	0.07033[4]	0.07522[9]	0.06945[3]	0.07482[8]	0.06614[2]	0.07185[5]	0.08387[12]	0.078[11]	0.07334[7]	0.10325[15]	0.07685[10]	0.09048[14]	0.07243[6]	0.08589[13]
	E_6	0.05172[5]	0.05034[3]	0.05227[6]	0.04935[2]	0.05305[7]	0.04835[1]	0.05075[4]	0.0585[11]	0.0595[12]	0.05521[8]	0.0824[15]	0.05552[10]	0.06753[14]	0.05537[9]	0.06477[13]
	$\sum$ *Ranks*	17[3]	29[4]	49[8.5]	13[2]	49[8.5]	8[1]	35[6]	75[12.5]	64[11]	34[5]	90[15]	63[10]	83[14]	36[7]	75[12.5]
60	E_1	0.32501[3]	0.34068[5]	0.36157[8]	0.31485[2]	0.36621[9]	0.31455[1]	0.36096[6]	0.42213[12]	0.38109[11]	0.36143[7]	0.65402[15]	0.37305[10]	0.42999[13]	0.33212[4]	0.44092[14]
	E_2	0.17267[3]	0.18923[5]	0.21535[8]	0.1559[1]	0.21941[9]	0.15881[2]	0.21221[7]	0.2919[12]	0.23496[10]	0.20419[6]	0.87525[15]	0.23977[11]	0.3071[14]	0.17846[4]	0.30198[13]
	E_3	0.08125[3]	0.08517[5]	0.09039[8]	0.07871[2]	0.09155[9]	0.07864[1]	0.09024[6]	0.10553[12]	0.09527[11]	0.09036[7]	0.16351[15]	0.09326[10]	0.1075[13]	0.08303[4]	0.11023[14]
	E_4	0.0263[2]	0.02775[5]	0.0293[6]	0.0265[3]	0.02975[8]	0.02588[1]	0.02935[7]	0.03388[12]	0.03177[11]	0.0305[10]	0.04807[15]	0.03012[9]	0.03606[13]	0.02755[4]	0.03746[14]
	E_5	0.03852[2]	0.04072[5]	0.04313[7]	0.03871[3]	0.04364[8]	0.03781[1]	0.04308[6]	0.04987[12]	0.04636[11]	0.04456[10]	0.07132[15]	0.04425[9]	0.05294[13]	0.04034[4]	0.05482[14]
	E_6	0.02441[2]	0.0253[5]	0.02616[7]	0.0245[3]	0.02593[6]	0.02435[1]	0.02504[4]	0.0296[11]	0.03[12]	0.02788[9]	0.04347[15]	0.02813[10]	0.03319[13]	0.02748[8]	0.03336[14]
	$\sum$ *Ranks*	15[3]	30[5]	44[7]	14[2]	49[8.5]	7[1]	36[6]	71[12]	66[11]	49[8.5]	90[15]	59[10]	79[13]	28[4]	83[14]
120	E_1	0.22134[1]	0.24378[4]	0.25882[8]	0.22356[3]	0.25969[9]	0.22252[2]	0.24534[5]	0.30446[13]	0.27368[10]	0.25609[7]	0.47753[15]	0.27801[11]	0.32414[14]	0.24931[6]	0.29993[12]
	E_2	0.0791[2]	0.09555[4]	0.10772[8]	0.07861[1]	0.10808[9]	0.08135[3]	0.0959[5]	0.14944[13]	0.12215[10]	0.10473[7]	0.46961[15]	0.124[11]	0.16973[14]	0.09683[6]	0.14004[12]
	E_3	0.05533[1]	0.06094[4]	0.06471[8]	0.05589[3]	0.06492[9]	0.05563[2]	0.06133[5]	0.07612[13]	0.06842[10]	0.06402[7]	0.11938[15]	0.0695[11]	0.08103[14]	0.06233[6]	0.07498[12]
	E_4	0.01811[1]	0.01994[4]	0.02131[7]	0.0187[3]	0.02135[8]	0.01831[2]	0.02012[5]	0.02462[12]	0.02268[10]	0.0214[9]	0.03654[15]	0.02271[11]	0.02699[14]	0.02068[6]	0.02516[13]
	E_5	0.02652[1]	0.02926[4]	0.03124[7]	0.02739[3]	0.03133[8]	0.02683[2]	0.0295[5]	0.03624[12]	0.03327[10]	0.03136[9]	0.05382[15]	0.03336[11]	0.03964[14]	0.0303[6]	0.03692[13]
	E_6	0.01582[3]	0.01614[4]	0.01696[7]	0.01548[2]	0.01665[6]	0.0152[1]	0.01617[5]	0.01888[11]	0.01953[12]	0.01837[10]	0.02942[15]	0.01791[9]	0.02128[13]	0.01771[8]	0.02162[14]
	$\sum$ *Ranks*	9[1]	24[4]	45[7]	15[3]	49[8.5]	12[2]	30[5]	74[12]	62[10]	49[8.5]	90[15]	64[11]	83[14]	38[6]	76[13]
200	E_1	0.17733[2]	0.17758[3]	0.19545[8]	0.1754[1]	0.19798[9]	0.17801[4]	0.18406[6]	0.21944[12]	0.20281[10]	0.19354[7]	0.35012[15]	0.2051[11]	0.23374[14]	0.18383[5]	0.23069[13]
	E_2	0.05034[4]	0.05031[3]	0.0592[7]	0.04799[1]	0.061[9]	0.05018[2]	0.05524[6]	0.07653[12]	0.06839[11]	0.0593[8]	0.26821[15]	0.06796[10]	0.08813[14]	0.05362[5]	0.08606[13]
	E_3	0.04433[2]	0.04439[3]	0.04886[8]	0.04385[1]	0.04949[9]	0.0445[4]	0.04602[6]	0.05486[12]	0.0507[10]	0.04838[7]	0.08753[15]	0.05127[11]	0.05844[14]	0.04596[5]	0.05767[13]
	E_4	0.01456[2]	0.01463[4]	0.01608[7]	0.01454[1]	0.01628[9]	0.01461[3]	0.01516[5]	0.01809[12]	0.01679[10]	0.01615[8]	0.02717[15]	0.01692[11]	0.01931[14]	0.01537[6]	0.01913[13]
	E_5	0.02137[1]	0.02147[3.5]	0.02359[7]	0.02138[2]	0.02394[9]	0.02147[3.5]	0.02222[5]	0.0265[12]	0.02462[10]	0.02371[8]	0.04014[15]	0.02485[11]	0.0284[14]	0.02258[6]	0.02812[13]
	E_6	0.01144[2]	0.01153[3]	0.01216[6]	0.01157[4]	0.01221[7]	0.01112[1]	0.01174[5]	0.01345[11]	0.01413[12]	0.01325[10]	0.02046[15]	0.01305[9]	0.01546[14]	0.01289[8]	0.01532[13]
	$\sum$ *Ranks*	13[2]	19.5[4]	43[7]	10[1]	52[9]	17.5[3]	33[5]	71[12]	63[10.5]	48[8]	90[15]	63[10.5]	84[14]	35[6]	78[13]
300	E_1	0.13803[1]	0.15481[4]	0.17056[9]	0.14269[2]	0.15612[5]	0.14499[3]	0.15848[6]	0.17645[12]	0.17137[10]	0.1633[8]	0.28153[15]	0.1728[11]	0.19302[13]	0.16145[7]	0.20187[14]
	E_2	0.02949[1]	0.03768[4]	0.0459[9]	0.03248[2]	0.03867[5]	0.03264[3]	0.03898[6]	0.04902[12]	0.04699[10]	0.04293[8]	0.14718[15]	0.04701[11]	0.05993[13]	0.04031[7]	0.06448[14]
	E_3	0.03451[1]	0.0387[4]	0.04264[9]	0.03567[2]	0.03903[5]	0.03625[3]	0.03962[6]	0.04411[12]	0.04284[10]	0.04083[8]	0.07038[15]	0.0432[11]	0.04825[13]	0.04036[7]	0.05047[14]
	E_4	0.01138[1]	0.0127[4]	0.01402[9]	0.01183[2]	0.01292[5]	0.01192[3]	0.01301[6]	0.01444[12]	0.01415[10]	0.01351[8]	0.02235[15]	0.01422[11]	0.01608[13]	0.01341[7]	0.01674[14]
	E_5	0.01669[1]	0.01868[4]	0.02061[9]	0.01736[2]	0.01895[5]	0.01751[3]	0.01914[6]	0.02124[12]	0.0208[10]	0.01988[8]	0.03294[15]	0.0209[11]	0.02364[13]	0.01968[7]	0.02459[14]
	E_6	0.00887[3]	0.00915[4]	0.00926[6]	0.00875[2]	0.00929[7]	0.00872[1]	0.00917[5]	0.01062[11]	0.01112[12]	0.01056[10]	0.01589[15]	0.00983[8]	0.01195[13]	0.01011[9]	0.01202[14]
	$\sum$ *Ranks*	8[1]	24[4]	51[9]	12[2]	32[5]	16[3]	35[6]	71[12]	62[10]	50[8]	90[15]	63[11]	78[13]	44[7]	84[14]
400	E_1	0.1194[1]	0.12781[5]	0.13447[6]	0.12061[2]	0.13478[7]	0.12335[4]	0.12252[3]	0.15588[12]	0.15056[11]	0.13762[10]	0.23294[15]	0.13652[9]	0.16062[13]	0.13607[8]	0.16538[14]
	E_2	0.02303[2]	0.02636[5]	0.02783[6]	0.02292[1]	0.02887[7]	0.02382[3]	0.0243[4]	0.03884[12]	0.03572[11]	0.02946[8]	0.10393[15]	0.02986[10]	0.04189[13]	0.02949[9]	0.04399[14]
	E_3	0.02985[1]	0.03195[5]	0.03362[6]	0.03015[2]	0.0337[7]	0.03084[4]	0.03063[3]	0.03897[12]	0.03764[11]	0.03441[10]	0.05824[15]	0.03413[9]	0.04015[13]	0.03402[8]	0.04134[14]
	E_4	0.0098[1]	0.01049[5]	0.01109[6]	0.00999[2]	0.0111[7]	0.01017[4]	0.01011[3]	0.01279[12]	0.0125[11]	0.01142[10]	0.01851[15]	0.01128[9]	0.01332[13]	0.01127[8]	0.01371[14]
	E_5	0.01438[1]	0.01543[5]	0.01629[6]	0.01469[2]	0.01631[7]	0.01493[4]	0.01485[3]	0.01883[12]	0.01834[11]	0.01677[10]	0.0273[15]	0.01655[8]	0.01957[13]	0.01656[9]	0.02013[14]
	E_6	0.00736[2]	0.00766[5]	0.00785[6]	0.00738[3]	0.00788[7]	0.0073[1]	0.00753[4]	0.00876[11]	0.00925[12]	0.00857[10]	0.01321[15]	0.00834[8.5]	0.01001[14]	0.00834[8.5]	0.00998[13]
	$\sum$ *Ranks*	8[1]	30[5]	36[6]	12[2]	42[7]	20[3.5]	20[3.5]	71[12]	67[11]	58[10]	90[15]	53.5[9]	79[13]	50.5[8]	83[14]

Table 10. Numerical values for simulation measures for (α = 4.0) under RSS.

n		MLE	ADE	CVME	MPSE	OLSE	R-ADE	WLSE	L-ADE	MNSADE	MNSALODE	AD-LSOE	KE	MNSSDE	MNSSLOD	MNSLXDE
20	E_1	0.34626[1]	0.35091[5]	0.37561[6]	0.38254[8]	0.34781[2]	0.35028[4]	0.34954[3]	0.40216[9]	0.50229[12]	0.48956[11]	0.81354[15]	0.38155[7]	0.5895[13]	0.43998[10]	0.58999[14]
	E_2	0.19081[2]	0.19065[1]	0.22912[7]	0.22583[6]	0.20264[5]	0.20096[4]	0.19651[3]	0.26985[9]	0.41124[12]	0.37194[11]	1.28245[15]	0.24363[8]	0.56124[13]	0.30727[10]	0.56313[14]
	E_3	0.08657[1]	0.08773[5]	0.0939[6]	0.09564[8]	0.08695[2]	0.08757[4]	0.08738[3]	0.10054[9]	0.12557[12]	0.12239[11]	0.20338[15]	0.09539[7]	0.14738[13]	0.11[10]	0.1475[14]
	E_4	0.02815[1]	0.02868[2.5]	0.03078[6]	0.03297[9]	0.02877[4]	0.0288[5]	0.02868[2.5]	0.0323[8]	0.04167[11]	0.04206[12]	0.05766[15]	0.03101[7]	0.04995[13]	0.03658[10]	0.05027[14]
	E_5	0.04095[1]	0.04234[5]	0.04489[6]	0.04855[9]	0.042[2]	0.04229[4]	0.04216[3]	0.04728[8]	0.0613[11]	0.06175[12]	0.085[15]	0.04525[7]	0.07356[13]	0.05416[10]	0.07419[14]
	E_6	0.04763[5]	0.04706[3]	0.04865[7]	0.04698[2]	0.0483[6]	0.04541[1]	0.04734[4]	0.05246[9]	0.0559[12]	0.05325[11]	0.0771[15]	0.05259[10]	0.06189[13]	0.05202[8]	0.06433[14]
	$\sum$ *Ranks*	11[1]	21.5[4]	38[6]	42[7]	21[3]	22[5]	18.5[2]	52[9]	70[12]	68[11]	90[15]	46[8]	78[13]	58[10]	84[14]
60	E_1	0.19334[1]	0.19685[2]	0.21971[6]	0.22206[7]	0.2069[5]	0.203[4]	0.20182[3]	0.24008[9]	0.31186[12]	0.29999[11]	0.51788[15]	0.22261[8]	0.3414[13]	0.25584[10]	0.36129[14]
	E_2	0.06042[2]	0.06034[1]	0.07476[6]	0.0764[7]	0.06747[5]	0.06362[4]	0.06264[3]	0.09062[9]	0.15284[12]	0.13915[11]	0.62094[15]	0.07791[8]	0.18838[13]	0.10761[10]	0.20244[14]
	E_3	0.04834[1]	0.04921[2]	0.05493[6]	0.05551[7]	0.05173[5]	0.05075[4]	0.05045[3]	0.06002[9]	0.07797[12]	0.075[11]	0.12947[15]	0.05565[8]	0.08535[13]	0.06396[10]	0.09032[14]
	E_4	0.01585[1]	0.01619[2]	0.018[6]	0.01884[8]	0.01697[5]	0.01684[4]	0.01664[3]	0.01968[9]	0.02594[12]	0.0252[11]	0.03792[15]	0.01829[7]	0.02885[13]	0.02161[10]	0.0305[14]
	E_5	0.02327[1]	0.02379[2]	0.02651[6]	0.02763[8]	0.02496[5]	0.02472[4]	0.02441[3]	0.02888[9]	0.03808[12]	0.03703[11]	0.05598[15]	0.02689[7]	0.04234[13]	0.03167[10]	0.04475[14]
	E_6	0.02304[2]	0.0234[4]	0.02418[7]	0.02347[5]	0.02382[6]	0.02235[1]	0.02321[3]	0.02601[10]	0.02841[12]	0.02677[11]	0.04065[15]	0.02535[8]	0.03108[13]	0.0257[9]	0.0315[14]
	$\sum$ *Ranks*	8[1]	13[2]	37[6]	42[7]	31[5]	21[4]	18[3]	55[9]	72[12]	66[11]	90[15]	46[8]	78[13]	59[10]	84[14]
120	E_1	0.13424[1]	0.13763[2]	0.14258[3]	0.15858[7]	0.15199[6]	0.14825[5]	0.14456[4]	0.16558[9]	0.21326[12]	0.19561[11]	0.36973[15]	0.1606[8]	0.24726[13]	0.19183[10]	0.25119[14]
	E_2	0.02877[1]	0.02982[2]	0.03284[4]	0.03857[7]	0.03642[6]	0.03445[5]	0.03267[3]	0.04407[9]	0.07216[12]	0.06013[11]	0.32523[15]	0.04054[8]	0.09658[13]	0.05681[10]	0.09813[14]
	E_3	0.03356[1]	0.03441[2]	0.03564[3]	0.03965[7]	0.038[6]	0.03706[5]	0.03614[4]	0.0414[9]	0.05332[12]	0.0489[11]	0.09243[15]	0.04015[8]	0.06181[13]	0.04796[10]	0.0628[14]
	E_4	0.01104[1]	0.01133[2]	0.01175[3]	0.01328[8]	0.01249[6]	0.01226[5]	0.01191[4]	0.01358[9]	0.01781[12]	0.01627[11]	0.02794[15]	0.01323[7]	0.02075[13]	0.01594[10]	0.02103[14]
	E_5	0.01622[1]	0.01666[2]	0.01728[3]	0.01952[8]	0.01833[6]	0.01799[5]	0.0175[4]	0.01995[9]	0.0262[12]	0.02391[11]	0.04117[15]	0.01942[7]	0.03061[13]	0.02342[10]	0.03082[14]
	E_6	0.01483[3]	0.01497[4.5]	0.01543[7]	0.01481[2]	0.01521[6]	0.01469[1]	0.01497[4.5]	0.01694[10]	0.01855[12]	0.01765[11]	0.02634[15]	0.01651[8]	0.02051[14]	0.01693[9]	0.01999[13]
	$\sum$ *Ranks*	8[1]	14.5[2]	23[3]	39[7]	36[6]	26[5]	23.5[4]	55[9]	72[12]	66[11]	90[15]	46[8]	79[13]	59[10]	83[14]
200	E_1	0.10555[1]	0.10821[2]	0.11519[7]	0.11472[5]	0.11486[6]	0.11088[3]	0.11253[4]	0.13169[9]	0.17952[12]	0.16608[11]	0.29233[15]	0.12013[8]	0.19276[13]	0.14827[10]	0.1957[14]
	E_2	0.01765[1]	0.0181[2]	0.02159[6]	0.02038[5]	0.02193[7]	0.01942[4]	0.01906[3]	0.02704[9]	0.05015[12]	0.04224[11]	0.22814[15]	0.02258[8]	0.05929[13]	0.03507[10]	0.0619[14]
	E_3	0.02639[1]	0.02705[2]	0.0288[7]	0.02868[5]	0.02871[6]	0.02772[3]	0.02813[4]	0.03292[9]	0.04488[12]	0.04152[11]	0.07308[15]	0.03003[8]	0.04819[13]	0.03707[10]	0.04893[14]
	E_4	0.0087[1]	0.00889[2]	0.00947[6]	0.0095[7]	0.00945[5]	0.00913[3]	0.00928[4]	0.01089[9]	0.01485[12]	0.01386[11]	0.02233[15]	0.00989[8]	0.01608[13]	0.01232[10]	0.01636[14]
	E_5	0.01279[1]	0.01308[2]	0.01393[6]	0.014[7]	0.01388[5]	0.01343[3]	0.0136[4]	0.01596[9]	0.02184[12]	0.02036[11]	0.03286[15]	0.01456[8]	0.02367[13]	0.01811[10]	0.024[14]
	E_6	0.01102[5]	0.01085[2]	0.01139[7]	0.01089[3]	0.01118[6]	0.01052[1]	0.01092[4]	0.01233[10]	0.01373[12]	0.01296[11]	0.01958[15]	0.01191[8]	0.01493[13.5]	0.0122[9]	0.01493[13.5]
	$\sum$ *Ranks*	10[1]	12[2]	39[7]	32[5]	35[6]	17[3]	23[4]	55[9]	72[12]	66[11]	90[15]	48[8]	78.5[13]	59[10]	83.5[14]
300	E_1	0.08526[1]	0.08888[4]	0.09596[6]	0.09684[7]	0.09342[5]	0.08808[3]	0.08753[2]	0.10314[9]	0.14362[12]	0.12672[11]	0.22223[15]	0.10189[8]	0.16038[13]	0.11995[10]	0.16251[14]
	E_2	0.0112[1]	0.01247[4]	0.01423[6]	0.01465[7]	0.01335[5]	0.01204[3]	0.0119[2]	0.01705[9]	0.03181[12]	0.02554[11]	0.13529[15]	0.01626[8]	0.04013[13]	0.02214[10]	0.04072[14]
	E_3	0.02131[1]	0.02222[4]	0.02399[6]	0.02421[7]	0.02335[5]	0.02202[3]	0.02188[2]	0.02578[9]	0.0359[12]	0.03168[11]	0.05556[15]	0.02547[8]	0.0401[13]	0.02999[10]	0.04063[14]
	E_4	0.00702[1]	0.00731[4]	0.00791[6]	0.00806[7]	0.00771[5]	0.00727[3]	0.0072[2]	0.00848[9]	0.01191[12]	0.01046[11]	0.01729[15]	0.00839[8]	0.01333[13]	0.00995[10]	0.01354[14]
	E_5	0.01032[1]	0.01075[4]	0.01162[6]	0.01182[7]	0.01132[5]	0.01067[3]	0.01059[2]	0.01245[9]	0.01748[12]	0.0154[11]	0.02546[15]	0.01232[8]	0.01958[13]	0.01461[10]	0.0199[14]
	E_6	0.00853[5]	0.00848[3]	0.00877[7]	0.00849[4]	0.00876[6]	0.0081[1]	0.00847[2]	0.00959[10]	0.01064[12]	0.00981[11]	0.01456[15]	0.0093[8]	0.01144[13]	0.00947[9]	0.01151[14]
	$\sum$ *Ranks*	10[1]	23[4]	37[6]	39[7]	31[5]	16[3]	12[2]	55[9]	72[12]	66[11]	90[15]	48[8]	78[13]	59[10]	84[14]
400	E_1	0.07747[3]	0.07345[1]	0.08076[5]	0.0832[7]	0.08172[6]	0.07677[2]	0.08022[4]	0.08981[9]	0.12693[12]	0.11036[11]	0.19123[15]	0.08354[8]	0.13899[14]	0.09924[10]	0.13677[13]
	E_2	0.00917[2]	0.00864[1]	0.00998[4]	0.01071[7]	0.01065[6]	0.0094[3]	0.01006[5]	0.01287[9]	0.02494[12]	0.01955[11]	0.10206[15]	0.0114[8]	0.03155[14]	0.01504[10]	0.02987[13]
	E_3	0.01937[3]	0.01836[1]	0.02019[5]	0.0208[7]	0.02043[6]	0.01919[2]	0.02006[4]	0.02245[9]	0.03173[12]	0.02759[11]	0.04781[15]	0.02089[8]	0.03475[14]	0.02481[10]	0.03419[13]
	E_4	0.00639[3]	0.00604[1]	0.00666[5]	0.0069[8]	0.00672[6]	0.00632[2]	0.00661[4]	0.00738[9]	0.01054[12]	0.00916[11]	0.01501[15]	0.00688[7]	0.01153[14]	0.00821[10]	0.01141[13]
	E_5	0.00938[3]	0.0089[1]	0.00978[5]	0.01014[8]	0.00989[6]	0.0093[2]	0.00972[4]	0.01085[9]	0.01548[12]	0.01344[11]	0.02208[15]	0.0101[7]	0.01694[14]	0.01207[10]	0.01674[13]
	E_6	0.00697[2]	0.00704[3]	0.0073[6]	0.00708[4]	0.00737[7]	0.00687[1]	0.00713[5]	0.00792[9]	0.00889[12]	0.00824[11]	0.01244[15]	0.00775[8]	0.00974[14]	0.00797[10]	0.0097[13]
	$\sum$ *Ranks*	16[3]	8[1]	30[5]	41[7]	37[6]	12[2]	26[4]	54[9]	72[12]	66[11]	90[15]	46[8]	84[14]	60[10]	78[13]

Table 11. Numerical values for MSE of SRS divided by MSE of RSS for all estimators.

n	MLE	ADE	CVME	MPSE	OLSE	R-ADE	WLSE	L-ADE	MNSADE	MNSALODE	AD-LSOE	KE	MNSSDE	MNSSLODE	MNSLXDE
								$\alpha = 0.3$							
20	3.32258	3.21311	3.14493	2.43056	3.40909	2.70312	2.85714	3.25316	1.90196	1.72500	1.32506	2.78750	1.68783	1.88542	1.45366
60	3.21053	3.10526	3.18182	2.59091	2.95652	2.47826	3.15000	3.28000	1.58929	1.71429	1.62130	2.82609	1.57576	1.89189	1.65152
120	2.70000	2.81818	3.09091	2.25000	3.00000	2.80000	3.00000	2.85714	1.64286	1.85000	1.40952	2.83333	1.52941	1.77778	1.51429
200	2.66667	3.00000	3.50000	2.42857	2.71429	2.83333	3.16667	2.62500	1.52941	1.76923	1.59184	2.71429	1.47619	1.72727	1.52381
300	2.66667	3.00000	3.50000	2.42857	2.71429	2.83333	3.16667	2.62500	1.52941	1.76923	1.59184	2.71429	1.47619	1.72727	1.52381
400	2.66667	3.00000	3.33333	2.66667	3.33333	2.66667	3.00000	2.75000	1.50000	1.66667	1.08000	2.25000	1.45455	1.50000	1.36364
								$\alpha = 0.9$							
20	2.97561	3.18092	3.47033	2.10039	3.48107	2.55780	3.20465	3.03268	1.99664	1.71115	1.21273	2.86961	1.41654	1.86177	1.60540
60	3.05584	3.09048	2.80952	2.13306	3.13596	2.83562	3.14078	2.57798	1.70483	1.59390	1.26977	3.12977	1.51659	2.03430	1.69677
120	2.71429	3.01887	2.78814	2.30579	2.95614	2.91509	2.92453	2.91275	1.56207	1.74757	1.32140	2.91406	1.55523	1.77604	1.58311
200	2.58333	3.14062	2.62857	2.21333	3.02899	2.41176	3.08065	2.97561	1.83553	1.47445	1.46109	2.85135	1.41071	1.85965	1.53738
300	2.58333	3.14062	2.62857	2.21333	3.02899	2.41176	3.08065	2.97561	1.83553	1.47445	1.46109	2.85135	1.41071	1.85965	1.53738
400	2.53125	3.03333	2.90909	2.10811	2.91429	2.63636	2.70588	2.79070	1.31579	1.50000	1.70609	2.50000	1.69792	1.96296	1.41739
								$\alpha = 1.5$							
20	2.75229	3.20928	3.36416	2.24500	3.30840	2.82786	3.17729	3.05253	1.66493	1.86963	1.20874	3.07882	1.50592	2.00000	1.73820
60	2.63733	3.06646	3.22715	2.02657	2.74936	2.94127	3.09159	3.09785	1.60139	1.81459	1.28589	2.84379	1.47940	1.82896	1.84710
120	2.83217	3.04403	3.04632	2.13500	3.05946	2.57831	3.38742	2.80610	1.51893	1.70953	1.13868	3.05038	1.57400	1.51391	1.73884
200	2.49239	2.79293	3.16990	2.25792	2.92417	2.38424	3.19072	2.85870	1.45863	1.70426	1.20113	2.80723	1.49616	1.70479	1.49521
300	2.49239	2.79293	3.16990	2.25792	2.92417	2.38424	3.19072	2.85870	1.45863	1.70426	1.20113	2.80723	1.49616	1.70479	1.49521
400	2.68132	2.76842	2.78899	2.43243	3.33684	2.89109	2.88235	2.96992	1.65217	1.64000	1.35046	2.77049	1.44548	1.85475	1.45994
								$\alpha = 2.5$							
20	3.31674	3.38824	2.95376	2.14928	3.06112	3.02933	2.84471	2.85922	1.52435	1.70630	1.31260	2.93529	1.64529	1.91833	1.50549
60	2.74089	3.36079	3.26408	2.19270	2.90451	2.54421	3.06133	3.06923	1.56901	1.62369	1.34704	2.93022	1.57323	1.72387	1.55818
120	2.50276	2.58303	3.11480	1.86313	3.44513	2.32967	3.15096	2.91944	1.38680	1.40401	1.29265	3.15132	1.55286	1.82219	1.47712
200	2.73760	3.05854	2.92829	2.22834	3.01592	2.76630	3.11058	2.81555	1.49042	1.58685	1.26248	2.54773	1.35150	1.61965	1.49578
300	2.73760	3.05854	2.92829	2.22834	3.01592	2.76630	3.11058	2.81555	1.49042	1.58685	1.26248	2.54773	1.35150	1.61965	1.49578
400	2.75958	3.05431	3.14837	2.11017	3.10210	2.75224	2.90823	2.52539	1.51077	1.73155	1.16657	2.86224	1.39560	1.71701	1.55114
								$\alpha = 4.0$							
20	2.88444	3.33013	3.08764	2.28043	3.53938	2.56544	3.38100	3.32903	1.75311	1.55815	1.20444	2.99947	1.58588	1.96443	1.48731
60	2.85783	3.13606	2.88055	2.04058	3.25196	2.49623	3.38777	3.22114	1.53729	1.46741	1.40956	3.07753	1.63022	1.65840	1.49170
120	2.74939	3.20423	3.28015	2.03811	2.96760	2.36139	2.93541	3.39097	1.69277	1.74173	1.44393	3.05871	1.75740	1.70445	1.42709
200	2.85212	2.77956	2.74201	2.35476	2.78158	2.58393	2.89822	2.83025	1.36371	1.40388	1.17564	3.00974	1.48642	1.52894	1.39031
300	2.85212	2.77956	2.74201	2.35476	2.78158	2.58393	2.89822	2.83025	1.36371	1.40388	1.17564	3.00974	1.48642	1.52894	1.39031
400	2.51145	3.05093	2.78858	2.14006	2.71080	2.53404	2.41551	3.01787	1.43224	1.50691	1.01832	2.61930	1.32773	1.96077	1.47272

- It seems that the ML approaches are very useful for evaluating the estimated quality of SRS and RSS.
- RSS is more efficient than SRS, showing that RSS is a better sampling strategy with lower MSE and other metrics.

7. Real Data Analysis

A real dataset was judiciously selected to showcase the usefulness of the suggested estimation methods, and a comprehensive explanation is provided in this section. The goal was to illustrate the situations and ways these estimation methods could be utilized by investigating the real dataset. This examination exemplifies how these estimation techniques can be leveraged and highlights their practical applicability, underscoring their efficacy and relevance for practical research endeavors and informed decision-making processes.

The dataset encompasses the lifespan of a fatigue fracture in Kevlar 373/epoxy subjected to constant pressure at 90% stress level until all samples had failed [60]. The data values are presented in Table 14. The data analysis in Table 15 thoroughly examines descriptive statistics and graphical representations in Figure 3. These visual aids, including violin plots, total time on test (TTT) plots, histograms, kernel density plots, quantile-quantile (Q-Q) plots, and box plots, offer valuable insights into the dataset's characteristics and distribution patterns.

Furthermore, a KS test was conducted to evaluate the dataset's compatibility with a C-JD. The results, with a KS distance of 0.1154 and a p-value of 0.2444, indicate the suitability of the chosen

Table 12. Partial and overall ranks for all estimation methods of our proposed model by SRS.

Parameter	n	MLE	ADE	CVME	MPSE	OLSE	R-ADE	WLSE	L-ADE	MNSADE	MNSALODE	AD-LSOE	KE	MNSSDE	MNSSLODE	MNSLXDE
$\alpha = 0.3$	20	3.5	5.0	8.0	3.5	7.0	1.0	2.0	11.0	12.0	9.0	15.0	10.0	14.0	6.0	13.0
	60	2.5	2.5	8.0	4.0	7.0	1.0	5.0	11.0	12.0	9.0	15.0	6.0	13.0	10.0	14.0
	120	1.0	4.0	7.0	2.0	5.0	3.0	6.0	11.0	12.0	10.0	15.0	9.0	13.0	8.0	14.0
	200	1.0	5.0	9.0	2.0	6.0	3.0	4.0	10.0	12.0	11.0	15.0	7.0	13.0	8.0	14.0
	300	4.0	1.0	7.0	3.0	6.0	5.0	2.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	400	2.0	5.0	9.0	3.0	6.0	1.0	4.0	10.0	12.0	11.0	15.0	8.0	14.0	7.0	13.0
$\alpha = 0.9$	20	2.0	4.0	7.0	1.0	8.0	3.0	6.0	11.0	13.0	9.0	15.0	10.0	12.0	5.0	14.0
	60	2.0	5.5	5.5	1.0	7.0	3.0	4.0	10.0	12.0	8.0	15.0	11.0	13.0	9.0	14.0
	120	1.0	5.0	6.0	2.0	7.0	3.0	4.0	11.0	12.0	10.0	15.0	9.0	13.0	8.0	14.0
	200	1.0	6.0	4.0	3.0	7.0	2.0	5.0	11.0	12.0	8.0	15.0	9.0	13.0	10.0	14.0
	300	2.0	5.0	6.0	1.0	7.0	3.0	4.0	11.0	12.0	10.0	15.0	9.0	14.0	8.0	13.0
	400	2.0	4.5	6.0	1.0	7.0	3.0	4.5	11.0	12.0	9.0	15.0	8.0	13.0	10.0	14.0
$\alpha = 1.5$	20	1.0	4.0	10.0	2.5	7.0	2.5	5.0	11.5	11.5	8.5	15.0	8.5	13.0	6.0	14.0
	60	2.0	4.0	7.0	1.0	6.0	3.0	5.0	11.0	12.0	10.0	15.0	9.0	13.0	8.0	14.0
	120	1.0	4.0	7.0	2.0	8.0	3.0	6.0	11.0	12.0	9.0	15.0	10.0	13.0	5.0	14.0
	200	2.0	4.0	8.0	3.0	5.0	1.0	6.0	11.0	12.0	9.0	15.0	10.0	14.0	7.0	13.0
	300	2.0	5.0	6.0	1.0	7.0	3.0	4.0	11.0	12.0	10.0	15.0	9.0	13.0	8.0	14.0
	400	1.0	2.0	6.0	3.0	7.0	4.0	5.0	11.0	12.0	8.0	15.0	9.0	13.0	10.0	14.0
$\alpha = 2.5$	20	3.0	4.0	5.0	1.5	8.0	1.5	7.0	12.0	10.5	9.0	15.0	10.5	14.0	6.0	13.0
	60	2.0	6.0	8.0	3.0	7.0	1.0	4.0	12.0	11.0	9.0	15.0	10.0	14.0	5.0	13.0
	120	1.0	4.0	7.0	2.0	9.0	3.0	5.0	12.0	11.0	6.0	15.0	10.0	14.0	8.0	13.0
	200	2.0	4.0	9.5	1.0	8.0	3.0	5.0	11.0	12.0	7.0	15.0	9.5	13.0	6.0	14.0
	300	1.0	5.0	10.0	2.0	7.0	3.0	4.0	11.0	12.0	9.0	15.0	8.0	14.0	6.0	13.0
	400	2.0	4.0	8.0	1.0	7.0	5.0	3.0	11.0	12.0	10.0	15.0	9.0	13.0	6.0	14.0
$\alpha = 4.0$	20	3.0	4.0	8.5	2.0	8.5	1.0	6.0	12.5	11.0	5.0	15.0	10.0	14.0	7.0	12.5
	60	3.0	5.0	7.0	2.0	8.5	1.0	6.0	12.0	11.0	8.5	15.0	10.0	13.0	4.0	14.0
	120	1.0	4.0	7.0	3.0	8.5	2.0	5.0	12.0	10.0	8.5	15.0	11.0	14.0	6.0	13.0
	200	2.0	4.0	7.0	1.0	9.0	3.0	5.0	12.0	10.5	8.0	15.0	10.5	14.0	6.0	13.0
	300	1.0	4.0	9.0	2.0	5.0	3.0	6.0	12.0	10.0	8.0	15.0	11.0	13.0	7.0	14.0
	400	1.0	5.0	6.0	2.0	7.0	3.5	3.5	12.0	11.0	10.0	15.0	9.0	13.0	8.0	14.0
$\sum$ Ranks		55.0	128.5	218.5	61.5	212.5	77.5	141.0	335.0	348.5	267.5	450.0	278.0	400.0	218.0	408.5
Overall Rank		1	4	8	2	6	3	5	11	12	9	15	10	13	7	14

Table 13. Partial and overall ranks for all estimation methods of our proposed model by RSS.

Parameter	n	MLE	ADE	CVME	MPSE	OLSE	R-ADE	WLSE	L-ADE	MNSADE	MNSALODE	AD-LSOE	KE	MNSSDE	MNSSLODE	MNSLXDE
$\alpha = 0.3$	20	1.0	2.0	5.0	7.0	6.0	4.0	3.0	8.0	12.0	11.0	15.0	9.0	13.0	10.0	14.0
	60	2.0	1.0	3.0	7.0	6.0	5.0	4.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	120	1.0	2.0	4.0	8.0	6.0	3.0	5.0	9.0	11.0	12.0	15.0	7.0	13.0	10.0	14.0
	200	3.0	1.0	4.5	6.0	7.0	2.0	4.5	11.0	12.0	10.0	15.0	9.0	14.0	8.0	13.0
	300	1.0	2.0	4.0	7.0	6.0	5.0	3.0	9.0	10.0	13.0	15.0	8.0	12.0	11.0	14.0
	400	3.0	2.0	1.0	7.0	6.0	5.0	4.0	9.0	14.0	12.0	15.0	8.0	13.0	10.0	11.0
$\alpha = 0.9$	20	1.0	2.0	3.5	7.0	6.0	5.0	3.5	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	60	1.0	3.0	6.0	7.0	5.0	4.0	2.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	120	1.0	2.0	6.0	7.0	5.0	3.0	4.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	200	3.0	2.0	8.0	7.0	5.0	4.0	1.0	9.0	12.0	11.0	15.0	6.0	14.0	10.0	13.0
	300	4.0	1.0	5.0	6.0	7.0	3.0	2.0	9.0	12.0	11.0	15.0	8.0	14.0	10.0	13.0
	400	1.0	3.0	4.0	6.5	6.5	2.0	5.0	8.0	11.0	12.0	15.0	9.0	13.0	10.0	14.0
$\alpha = 1.5$	20	1.0	2.0	5.0	7.0	6.0	3.0	4.0	9.0	12.0	11.0	15.0	8.0	14.0	10.0	13.0
	60	4.0	1.0	5.0	7.0	6.0	2.0	3.0	9.0	12.0	11.0	15.0	8.0	14.0	10.0	13.0
	120	1.0	3.0	5.0	8.0	6.0	4.0	2.0	9.0	12.0	11.0	15.0	7.0	14.0	10.0	13.0
	200	2.0	3.0	5.0	7.0	6.0	4.0	1.0	9.0	12.0	11.0	15.0	8.0	14.0	10.0	13.0
	300	1.0	2.0	6.0	7.0	3.0	4.0	5.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	400	1.0	3.0	6.5	6.5	2.0	4.0	5.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
$\alpha = 2.5$	20	2.0	1.0	4.0	7.0	6.0	3.0	5.0	9.0	12.0	11.0	15.0	8.0	14.0	10.0	13.0
	60	2.0	1.0	5.0	7.0	6.0	4.0	3.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	120	2.0	3.0	4.0	8.0	5.0	6.0	1.0	9.0	12.0	11.0	15.0	7.0	13.0	10.0	14.0
	200	3.0	1.0	7.0	6.0	5.0	4.0	2.0	9.0	12.0	11.0	15.0	8.0	14.0	10.0	13.0
	300	1.0	3.0	5.0	6.0	8.0	2.0	4.0	9.0	12.0	11.0	15.0	7.0	14.0	10.0	13.0
	400	1.0	2.0	4.0	7.0	5.5	5.5	3.0	9.0	12.0	11.0	15.0	8.0	14.0	10.0	13.0
$\alpha = 4.0$	20	1.0	4.0	6.0	7.0	3.0	5.0	2.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	60	1.0	2.0	6.0	7.0	5.0	4.0	3.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	120	1.0	2.0	3.0	7.0	6.0	5.0	4.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	200	1.0	2.0	7.0	5.0	6.0	3.0	4.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	300	1.0	4.0	6.0	7.0	5.0	3.0	2.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	400	3.0	1.0	5.0	7.0	6.0	2.0	4.0	9.0	12.0	11.0	15.0	8.0	14.0	10.0	13.0
$\sum$ Ranks		51.0	63.0	148.5	206.0	167.0	112.5	98.0	270.0	358.0	334.0	450.0	237.0	401.0	299.0	405.0
Overall Rank		1	2	5	7	6	4	3	9	12	11	15	8	13	10	14

Table 14. The life of fatigue fracture of Kevlar 373/epoxy subjected to constant pressure at 90% stress level until all had failed.

0.0251	0.0886	0.0891	0.2501	0.3113	0.3451	0.4763	0.5650	0.5671	0.6566	0.6748
0.6751	0.6753	0.7696	0.8375	0.8391	0.8425	0.8645	0.8851	0.9113	0.9120	0.9836
1.0483	1.0596	1.0773	1.1733	1.2570	1.2766	1.2985	1.3211	1.3503	1.3551	1.4595
1.4880	1.5728	1.5733	1.7083	1.7263	1.7460	1.7630	1.7746	1.8475	1.8375	1.8503
1.8808	1.8878	1.8881	1.9316	1.9558	2.0048	2.0408	2.0903	2.1093	2.1330	2.2100
2.2460	2.2878	2.3203	2.3470	2.3513	2.4951	2.5260	2.9911	3.0256	3.2678	3.4045
3.4846	3.7433	3.7455	3.9143	4.8073	5.4005	5.4435	5.5295	6.5541	9.0960	

distribution for modeling the real dataset. Figure 4 affirms these results.
In light of the theoretical results and discourse offered earlier, the dataset was analyzed using two sampling techniques: SRS and RSS. Tables 16 and 17 showcase the estimates obtained from SRS and RSS applied to the C-JD. These estimates cover a range of sample sizes and estimation techniques, providing a comprehensive view of the results achieved through different sampling methods and estimation procedures. To underscore the advantages of RSS compared to SRS across the various estimation techniques, we conducted an assessment using multiple goodness-of-fit statistics, including the AD test statistic (A^*), CVM (C^*), and KS test (K^*). These statistical tests were employed to evaluate how well the data aligns with the model, offering insights into how effectively RSS captures the underlying distribution compared to SRS. Table 18 presents a comparative analysis of the goodness-of-fit values between the SRS and RSS designs. The comparative analysis between the two designs, utilizing goodness-of-fit values, facilitates the assessment of their relative effectiveness in fitting the dataset to the proposed model. This comparison identifies which sampling design and estimation techniques yield superior goodness-of-fit results. The model's fit to the dataset is visualized in Figures 6 and 7. Notably, the RSS design exhibits better performance than the SRS design regarding efficiency, as evidenced by the smaller goodness-of-fit values obtained. This superiority of RSS over SRS is consistently observed across all estimates. These findings highlight the advantages of utilizing RSS over SRS for fitting the dataset to the model and obtaining more efficient estimates. The log-likelihood profile is presented in Figure 5.

Table 15. Conducting descriptive statistics and visualizations on the dataset.

	n	Mean	Median	Skewness	Kurtosis	Range	Minimum	Maximum	Sum
data	76	1.9595	1.7361	1.9791	8.1599	9.0709	0.0251	9.0960	148.9223

Table 16. Estimated parameter values of the C-JD, calculated through multiple estimation approaches utilizing the SRS dataset.

n	MLE	OLSE	WLSE	CVME	MPSE	ADE	RADE	L-ADE	MNSADE	MNSALODE	MNSSDE	MNSSLODE	MNSLXDE	KE	AD-LSOE
30	1.2169	1.1785	1.1619	1.1674	1.1792	1.1788	1.0529	0.9808	1.1405	1.1546	1.3012	1.1675	1.0791	1.1238	1.1330
50	1.2264	1.2084	1.1920	1.1955	1.1971	1.2035	1.1850	1.1055	1.0683	1.1634	1.0760	1.2972	1.0753	1.1641	1.1202
65	1.1517	1.1416	1.1352	1.1367	1.1258	1.1370	1.2885	1.2093	1.0951	1.1142	1.1815	1.1412	1.1829	1.1539	1.0090
75	1.1656	1.1685	1.1584	1.1598	1.1423	1.1603	1.2025	1.1333	1.2365	1.1240	1.2204	1.1517	1.2217	1.1817	1.0424

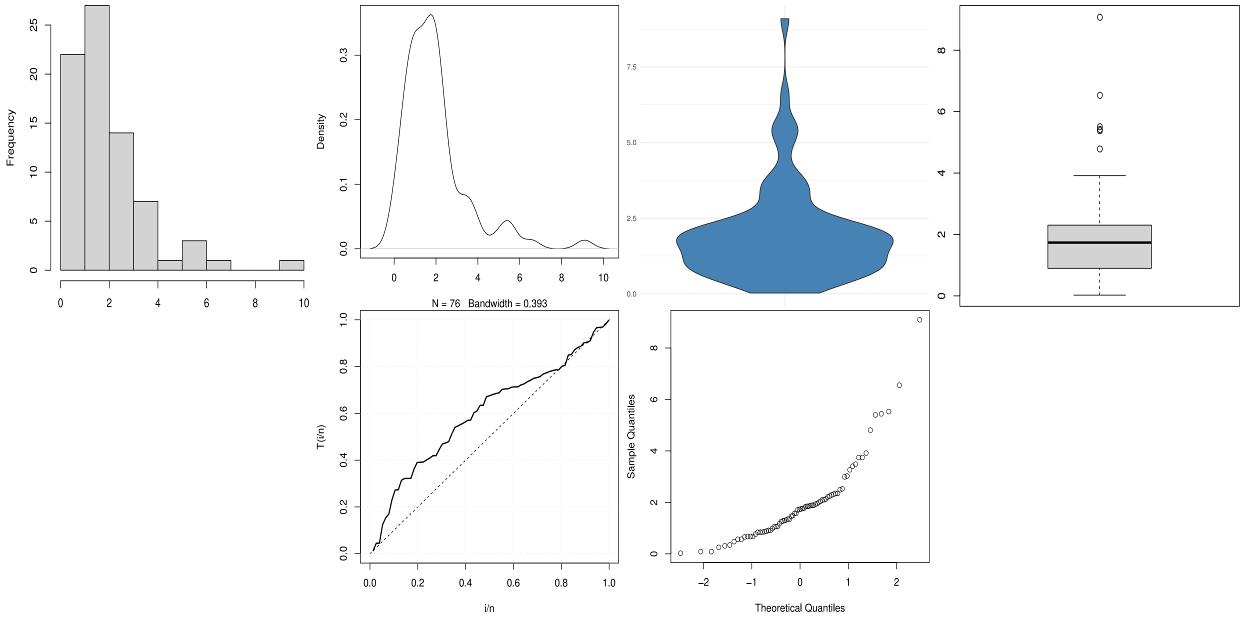

Figure 3. Graphical representation for the considered real dataset.

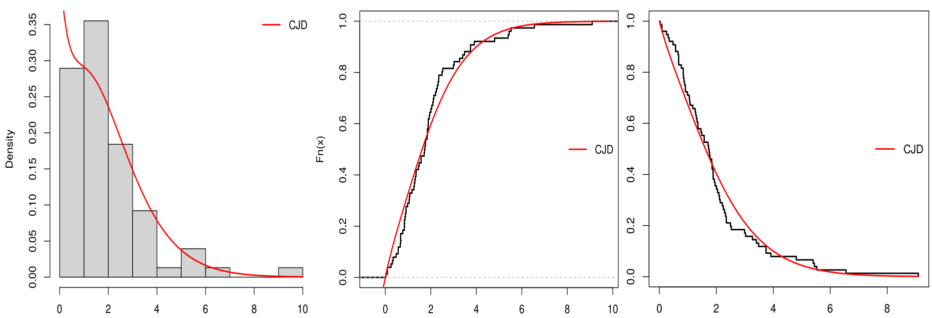

Figure 4. Graphical representation displaying the histogram overlaid with the estimated PDF, CDF, and survival function curves for the C-JD.

Table 17. Estimated parameter values of the C-JD, calculated through multiple estimation approaches utilizing the RSS dataset.

q	d	MLE	OLSE	WLSE	CVME	MPSE	ADE	R-ADE	L-ADE	MNSADE	MNSALODE	MNSSDE	MNSSLODE	MNSLXDE	KE	AD-LSOE
5	2	1.2946	1.2141	1.2358	1.2558	1.2710	1.2464	1.2785	1.2329	1.2790	1.2790	1.1899	1.2987	1.1873	1.2765	1.1830
5	3	1.1992	1.2058	1.1810	1.1890	1.1152	1.1808	1.2110	1.1022	0.9865	1.1271	1.1902	1.2011	0.9909	1.1957	1.3418
7	2	1.2025	1.2496	1.2407	1.2447	1.0476	1.2230	1.0767	1.1182	0.9258	0.9070	0.8916	0.9240	1.2891	1.1469	1.1528
7	3	1.2106	1.1914	1.2215	1.2289	1.2396	1.2043	1.1588	1.1089	1.3473	1.0856	1.5945	1.2342	1.5984	1.2816	1.0529

Table 18. The estimates and goodness-of-fit statistics (A^*,C^*, K^*) computed from the SRS and RSS design applied to the data, using a SRS sample of 10 units and RSS sample with $q = 5, d = 2$.

Method	design	Estimate	A^*	C^*	K^*
ML	SRS	0.8768	0.1725	1.0080	0.2871
	RSS	1.2348	0.1647	0.9662	0.1239
OLS	SRS	0.8421	0.1733	1.0126	0.3104
	RSS	1.1613	0.1661	0.9740	0.1210
WLS	SRS	0.8448	0.1732	1.0122	0.3085
	RSS	1.1722	0.1659	0.9728	0.1156
CVM	SRS	0.8662	0.1727	1.0094	0.2942
	RSS	1.1825	0.1657	0.9717	0.1105
MPS	SRS	0.8277	0.1737	1.0145	0.3202
	RSS	1.2291	0.1648	0.9668	0.1224
AD	SRS	0.8577	0.1729	1.0105	0.2998
	RSS	1.1762	0.1658	0.9724	0.1136
R-AD	SRS	1.0494	0.0385	0.3109	0.1978
	RSS	1.1405	0.0384	0.3105	0.1485
L-AD	SRS	0.9372	0.0385	0.3111	0.2649
	RSS	1.1232	0.0384	0.3106	0.1575
MNSAD	SRS	0.8962	0.1720	1.0054	0.2743
	RSS	1.2777	0.1639	0.9619	0.1354
MNSALOD	SRS	0.8962	0.1720	1.0054	0.2744
	RSS	1.2780	0.1639	0.9618	0.1355
MNSSD	SRS	0.7257	0.1763	1.0286	0.3922
	RSS	1.2157	0.1651	0.9682	0.1188
MNSSLOD	SRS	0.9526	0.1707	0.9983	0.2384
	RSS	1.2315	0.1648	0.9665	0.1230
MNSLXD	SRS	0.7169	0.1765	1.0298	0.3986
	RSS	1.2159	0.1651	0.9682	0.1189
Kolmogorov	SRS	0.8858	0.1722	1.0068	0.2811
	RSS	1.2317	0.1648	0.9665	0.1231
AD-LSO	SRS	0.9447	0.1708	0.9993	0.2433
	RSS	0.9937	0.1697	0.9932	0.2133

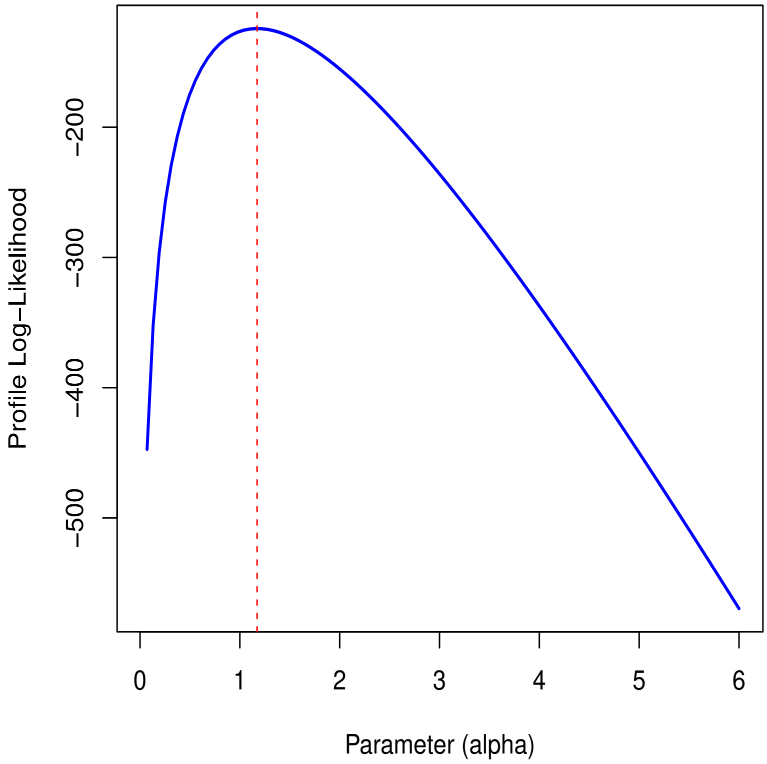

Figure 5. Profile log-likelihood plot for the C-JD.

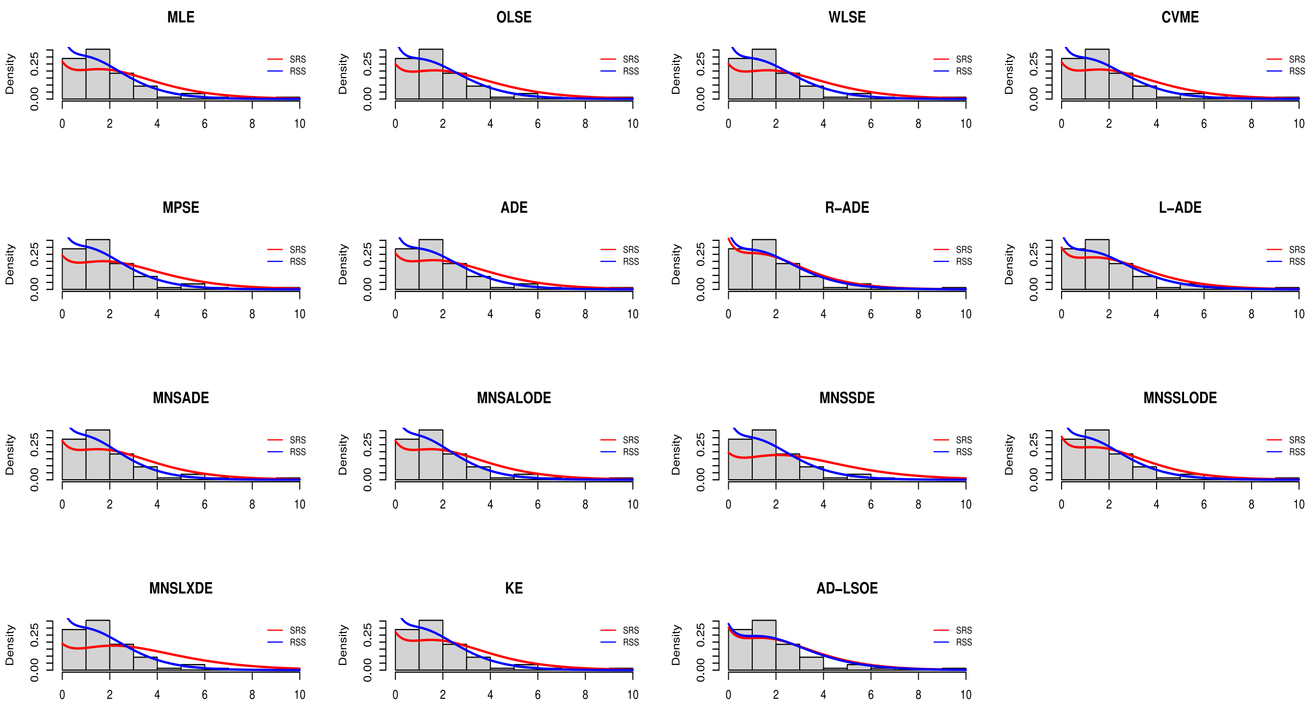

Figure 6. Plots of the estimated PDFs of the C-JD with histogram for the two sampling methods at n=10.

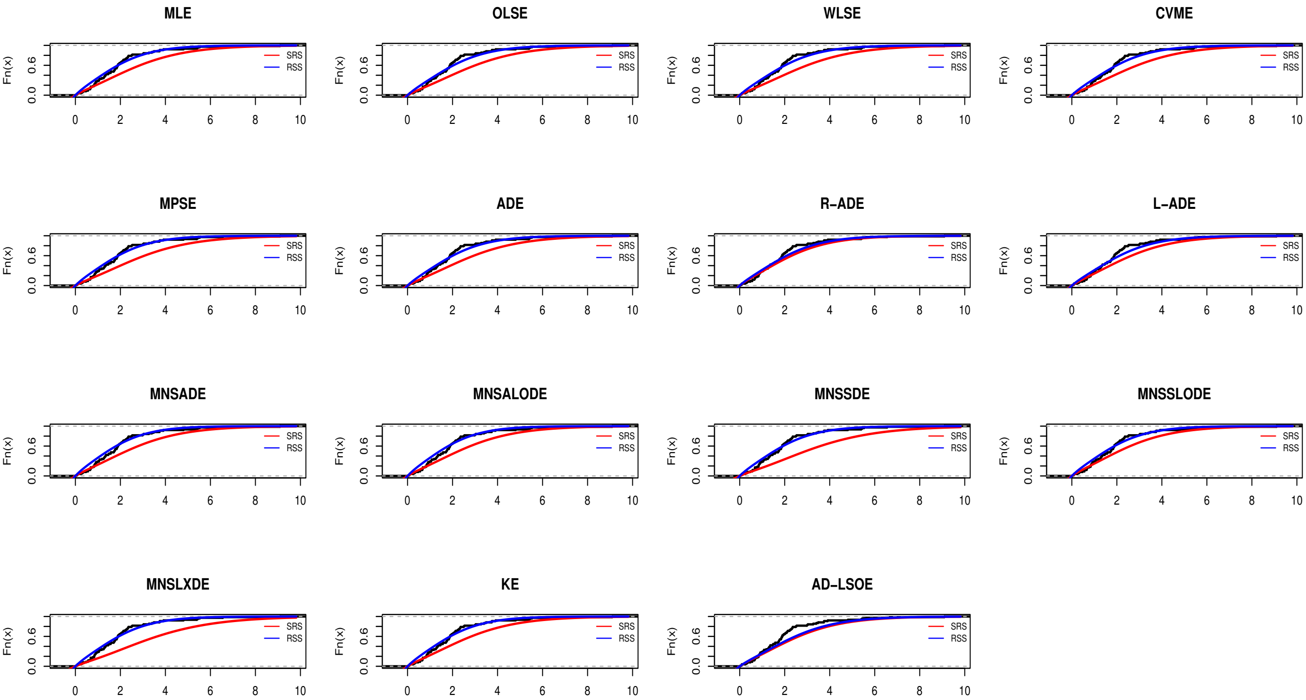

Figure 7. Plots of the estimated CDFs of the C-JD for the two sampling methods at n=10.

8. Summary and Conclusion

Efficient and economical sampling techniques are essential for agriculture, biology, environment, and ecology research. RSS is a valuable tool for achieving these goals while reducing costs. The C-JD is a novel distribution with a one-parameter that can be used to model various positive real-world datasets. This study aims to improve the C-JD's parameter estimate capabilities and accuracy under RSS design. No comparative analyses have been made between the fifteen proposed conventional estimating approaches for the C-JD parameter based on RSS. Among the well-known traditional estimating methods that are employed are the WLS, AD, ML, MNSAD, CVM, MNSSLOD, MPS, R-AD, OLS, MNSALOD, AD-LSO, MNSLXD, L-AD, MSSD, and KE. A comprehensive simulation research compares the efficiency of the obtained estimates. Utilizing several criteria measures, we do an extensive simulation study and evaluate the performance of different estimates based on RSS and SRS. Estimate methodologies and dataset evaluation will further narrow down to the best choice. Therefore, all estimates' partial and total ranks are found for both sampling procedures. The following conclusions are drawn from the numerical results:

- The ML approach consistently outperforms other methods, achieving overall scores of 51 and 55 for RSS and SRS datasets, respectively, in evaluating the quality of parameter estimates. Thus, we can affirm that ML is the best method for estimating the parameters of the C-JD, followed by the AD method.
- As sample sizes increase, estimates based on both SRS and RSS become highly reliable and converge toward the true values. RSS datasets consistently yield more reliable estimates than SRS datasets, as evidenced by our analysis.
- The RSS design outperforms the SRS strategy, as demonstrated by the analysis of the survival

data set.

A limitation of the research is that simulation studies are only considered in the scenario of perfect ranking. Future research may investigate the estimation of the C-JD in both perfect and imperfect ranking scenarios using modified RSS methods. Additionally, estimating the C-JD in stress-strength reliability models with outliers is another potential area of exploration [61, 62, 63, 64].

Availability of Data

Any data that supports the findings of this study is included in the article.

Conflict of Interest

The authors declare no competing interests.

Funding Statement

This work was supported and funded by the Deanship of Scientific Research at Imam Mohammad Ibn Saud Islamic University (IMSIU) (grant number IMSIU-DDRSP2503).

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