Computational Journal of Mathematical and Statistical Sciences
4(2), 407–423
DOI:10.21608/cjmss.2025.364215.1131
https://cjmss.journals.ekb.eg/

Research article

Improving geographically weighted Poisson regression model based on metaheuristic algorithms: Application to cancer rate data

Hind Mohammed Yousif[1], **Zakariya Yahya Algamal**[2,*]

[1] Department of Statistics and Informatics, University of Mosul, 41002 Mosul, Iraq; hind.23csp138@student.uomosul.edu.iq.

[2] Department of Statistics and Informatics, University of Mosul, 41002 Mosul, Iraq; zakariya.algamal@uomosul.edu.iq.

* **Correspondence:** zakariya.algamal@uomosul.edu.iq

Abstract: Geographically weighted Poisson regression (GWPR) model is a further refinishing of Poisson regression for model the spatial count data and consider local association of variables. Nevertheless, the GWPR model faces several challenges that can impact its effectiveness and reliability. One of these challenges is the bandwidth selection. An improper bandwidth value results to either fitting the GWPR model to the noise or output values that are unexpectedly low. A small bandwidth may well include too much local variability, while large bandwidth may average out important local changes. Meta-heuristic algorithms can be defined as optimization methods that designs up approximate solutions to problems, which involves searching through the solution space in the best way possible. Employment of meta-heuristic algorithms in determining the bandwidth value in GWPR model is entirely novel owing to the utilization of optimization methods in the selection of bandwidth value. In this paper, beluga whale optimization algorithm as meta-heuristic algorithms are employed to find the best value of the GWPR model bandwidth by considering the objective function for bandwidth selection as minimizing prediction errors. Based on cancer rate estimation as a real data application, the comparison studies and evaluations demonstrated that the proposed method outperformed other methods regarding pseudo-R2 and Deviance. According to the results, utilizing the meta-heuristic algorithms for estimating the bandwidth value in GWPR model presents a promising approach that combines advanced optimization techniques with spatial analysis.

Keywords: Geographically weighted Poisson regression, cancer rate, beluga whale optimization algorithm, bandwidth selection, kernel function.

Mathematics Subject Classification: 37J39, 62G08.

Received: 27 February 2025; Revised: 30 April 2025; Accepted: 12 May 2025; Online: 15 May 2025.

1. Introduction

Spatial data can be described as collected phenomena having either inherent or stated locational referent. It involves both spatial data content and spatial data entities as well. The recorded observations that are spatially distributed are defined here as content data while the data that correspond to one spatial entity but change through time are defined as change data. Spatial data objects are points, lines, areas, or surfaces with which content data are related.

Geographically modeling is a technique that provides much insight in the spatial analysis when relationships between variables differ from place to place [24, 32]. This method can be used to get closer to understanding how different phenomena change within different geographic locations. Spatial regression models are one of the biggest areas in geographically modeling. Spatial regression models are most useful to researchers who are trying to either spatially condition a dependent or independent variable, or are trying to model a spatial data set where there is excessive spatial autocorrelation which violates the assumptions of a classical regression analysis [20, 33]. Spatial regression models are used in several disciplines and areas for instance in economic, urban and regional planning, environmental conservation and health among others [17, 22, 30, 31, 35]. In addition, the spatial lag model can be employed [2, 23].

Geographically weighted regression (GWR) model is a type of spatial regression models which is used when the response variable is following the normal distribution [11]. Count data are often employed in social, economic, and epidemiological investigation [7, 8, 9, 10]. The values in this type of data are positive integers. One distribution that is well suited for this kind of data is the Poisson distribution for more details on this distribution. The count values from the response variable along with zeroes and possibly explanatory variables are linked and estimated through the Poisson regression model. For spatial count data, the geographically weighted Poisson regression (GWPR) model has been used to fully capture spatial behavior of the count response variable and the related explanatory variables.

However, there are numerous factors that could affect its efficiency and credibility, which are the challenges that are characteristic for GWPR model. A major difficulty in computing GWPR model is the determination of the bandwidth value before performing the computation. Over the last few years, a large number of natural-inspired algorithms have been successfully presented and implemented as the methods for random searching of the optimal solutions of a variety of optimization problems.

In this paper, we put forward the natural-inspired optimization algorithm to estimate the values of the bandwidth in GWPR model. In our proposed approach, we will be in a position to select the best values with higher accuracy of predictive value. In real data application, the efficacy of our proposed approach is shown to be superior to other approaches.

2. The description of GWPR model

Count data are used frequently in many types of research fields from Epidemiology, social and economic investigations. The following type of data is positive integers. Poisson distribution is a familiar distribution that used in modeling such type of data. This permits the examination of Poisson regression (PR) model aimed at modeling the counts as the response variable and possibly the explanation variable.

Let y_i be the response variable and follows a Poisson distribution with mean ω_i, then the probability density function is defined as

$$f(y_i) = \frac{e^{-\omega_i}\omega_i^{y_i}}{y_i!}, \quad y_i = 0, 1, \ldots; \quad i = 1, 2, \ldots, n. \tag{2.1}$$

In a PR model, $\ln(\omega_i) = \mathbf{x}_i^T\boldsymbol{\beta}$ is expressed as a linear combination of explanatory variables $\mathbf{x}_i = (x_{i1}, ..., x_{ip})^T$and $\boldsymbol{\beta}$ is a $(p+1)\times 1$ vector of unknown regression coefficients. According to this, the PR model can be as:

$$\begin{aligned} y_i &= \exp(\mathbf{x}_i^T\boldsymbol{\beta}) \\ &= \exp(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_p x_{ip}) \end{aligned} \tag{2.2}$$

The most common method of estimating the coefficients of PR model is to use the maximum likelihood (ML) method. Given the assumption that the observations are independent, the log-likelihood function is defined as

$$\ell(\boldsymbol{\beta}) = \sum_{i=1}^{n}\left\{y_i\mathbf{x}_i^T\boldsymbol{\beta} - \exp(\mathbf{x}_i^T\boldsymbol{\beta}) - \ln y_i!\right\}. \tag{2.3}$$

The ML estimator is then obtained after computing the first derivative of the Eq. (2.3) and setting it equal to zero. The ML estimator of PR model coefficients, $\hat{\boldsymbol{\beta}}_{PR}$ is

$$\hat{\boldsymbol{\beta}}_{PR} = (\mathbf{X}^T\hat{\mathbf{Z}}\mathbf{X})^{-1}\mathbf{X}^T\hat{\mathbf{Z}}\hat{\mathbf{v}}, \tag{2.4}$$

where $\hat{\mathbf{Z}} = \mathrm{diag}(\hat{\omega}_i)$ and $\hat{\mathbf{v}}$ is a vector where ith element equals to $\hat{v}_i = \ln(\hat{\omega}_i) + ((y_i - \hat{\omega}_i)/\hat{\omega}_i)$.

In practice, the relationships between variables might vary geographically. Unlike global regression (PR model), where the regression coefficients that arise in PR are fixed over space. GWPR model enables local variations in the estimation of coefficients [6, 13]. In other words, the coefficients are estimate locally at spatial references data points using GWPR model. The GWR model is defined as [12, 24].

$$\begin{aligned} y_{i,spatial} &= \exp(\mathbf{x}_i^T\boldsymbol{\beta}(r_i, q_i)) \\ &= \exp(\beta_0(r_i, q_i) + \beta_1(r_i, q_i)x_{i1} + \beta_2(r_i, q_i)x_{i2} + ... + \beta_p(r_i, q_i)x_{ip}) \end{aligned} \tag{2.5}$$

where $\beta_j(r_i, q_i),\ \ j = 1, 2, ..., p$ is the coefficients which are varying conditionals on the location and $(r_i, q_i)$is the two-dimensional coordinates of the ith point in the geographical location. Thus, the spatial heterogeneity is handled by GWPR model in a way that permits by parameters to be location dependent, thus enables estimation of localized effects.

Based on locally weighted likelihood method which is maximizing the geographically weighted log-likelihood function, the estimated coefficient, $\hat{\boldsymbol{\beta}}_{GWPR}$, at location i, can be obtained as

$$\hat{\boldsymbol{\beta}}_{GWPR} = (\mathbf{X}^T\mathbf{W}(r_i, q_i)\mathbf{X})^{-1}\mathbf{X}^T\mathbf{W}(r_i, q_i)\mathbf{y}, \tag{2.6}$$

where $\mathbf{W}(r_i, q_i)$is an $n \times n$spatial weight matrix. Spatial weights are quantitative measures associated with observations to derive them based on the distance from the focal observation. They define how the observations within a close working range impact the auto regression characteristics of the prediction for the particular position. In GWPR model, these weights are obtained through a kernel function which describes the relative position between the data points. Several kernel functions available for weighting and were adopted for use in developing the GWPR model such as box-car, bi-square, tri-cube, exponential, Gaussian among them [16, 21, 33, 37].

3. Bandwidth selection

In general, the GWPR modeling involves estimating for each intersection a regression equation supported by the observations in other intersections. Experience has shown that observations near intersection *i*will contribute more to the estimation of the parameters of i than would those far from it. This impact reduces as the distance between the two places rises gradually. For estimating the smoothed geographical variations in the parameters with a distance based weighting scheme, GWPR model uses a spatial kernel method [14].

The bandwidth of the kernel, up to which the weight is assigned, could be pre-specified based on the distance or post-specified based on a certain number of neighbours. In a similar manner, the modeling outcomes of the GWPR model are significant in the selection of the bandwidth [34]. A relatively small bandwidth that only contains a few observations can lead to instability in the fits while a conversely large value for the bandwidth can lead to bias [15].

The bandwidth parameter determines the size of the neighborhood taken into consideration by each observation in weighing. When the bandwidth is small it can incorporate local changes, while if it is large, it could overcome these alterations. The adaptive bandwidth methods can themselves adapt to the density of data, this of course means that the methods can provide a more refined model[18, 19]. This selection can be done using either fixed selection, where a constant value applied across all observations, or adaptive selection, where varies values applied based on data density, allowing for more flexibility in capturing local variations. Several kernel functions are summarized in Table 1. As in Table 1, the kernel function assigns weights to observations based on their Euclidean distance, d_{ij}, from the regression point being estimated. In addition, the bandwidth, σ, which is representing the number of neighboring data, needs to be determined.

Table 1. Kernel functions in GWPR model

Kernel	Mathematical form
Gaussian	$w_{ij} = \exp\left(-\frac{1}{2}\left(\frac{d_{ij}}{\sigma}\right)^2\right)$
Exponential	$w_{ij} = \exp\left(-\frac{\lvert d_{ij}\rvert}{\sigma}\right)$
Bi-square	$w_{ij} = \begin{cases} \left(1-\left(\frac{d_{ij}}{\sigma}\right)^2\right)^2 & if\lvert d_{ij}\rvert < \sigma, \\ 0 & otherwise. \end{cases}$
Tri-cube	$w_{ij} = \begin{cases} \left(1-\left(\frac{d_{ij}}{\sigma}\right)^3\right)^3 & if\lvert d_{ij}\rvert < \sigma, \\ 0 & otherwise. \end{cases}$
Box-car	$w_{ij} = \begin{cases} 1 & if\lvert d_{ij}\rvert < \sigma, \\ 0 & otherwise. \end{cases}$

The idea behind estimating the bandwidth value in GWPR model is to determine the optimal extent of spatial influence that neighboring observations have on the regression estimates for a specific location. Bandwidth selection is crucial because it directly affects the model's ability to capture local variations in relationships between dependent and independent variables. Methods such as cross-validation (CV), generalized cross-validation (GCV), and information criteria like Akaike information criterion

(AIC) or corrected AIC (CAIC) can help identify the optimal bandwidth, σ[19, 23].

Meta-heuristic algorithm are the refined methods of optimization employed to find good solutions to problems which are hard to solve conventionally [9]. These algorithms are widely used where the solution space is large, non-linear, or not very well defined [28]. These algorithms are planned to come out of local optimal and aimed for global optima than local search hence more accurate than local searches. Further, the nature of these algorithms is that they can give good solutions at once especially in the search spaces of high dimensions, which can be hardly solved by the traditional optimization processes [1].

From this point, our proposed idea is to use meta-heuristic algorithms for estimating the bandwidth value in GWPR model which can offer a promising alternative to traditional methods. Through the application of these optimization techniques, our proposed idea is able to improve their probability of identifying optimal bandwidths that leads to accurate model representation and presentation. When the nature of spatial data analysis becomes more intricate, the implementation of complex optimization solutions to support modeling may be essential. In this paper, beluga whale optimization (BWO) algorithm [36], which is swarm-based metaheuristic algorithm inspired from the behaviors of beluga whales, is employed to tune the optimal bandwidth value in the GWPR model.

BWO algorithm basically replicates the behaviors of beluga whales which include group swimming, foraging as well as the whale falls by which a dead whale enriches other organisms in the seas. This biological inspiration enables BWO to invoke and discover the required solution space well. The algorithm operates through three key phases: Three are divisions that were identified namely E1 The Exploration phase, E2 the Exploitation phase, and E3 the Whale fall phase. As the first phase, beluga whales try to find food in relations to their surroundings, which is where the solution space is being searched for the best solutions. During the second stage, the solutions that the algorithm found may introduce certain potential problems which then are altered in order to present better solutions. While the third phase provides emulation of what happens after the death of a whale when resources in the ecosystem are reallocating; in this case, the solutions as per their performance. The following are the parameter combinations for our suggested methodology.

1. The number of beluga whales BWO algorithm is 15 members and the number of iterations is t_{max} = 500.
2. Every member's position is representing the bandwidth value of the kernel, σin Table 1 and it chosen at random. The members' starting positions are produced from a uniform distribution in the interval $[5, n]$where $n$represents the number of samples in the real data under the study.
3. The definition of the fitness function is considered as the deviance criterion and it is defined as

$$\text{fitness} = \min D\left(y; \hat{y}\left(\hat{\beta}_{GWPR}\right)\right) = 2\sum_{i=1}^{n}\left[y_i \log\left(\frac{y_i}{\hat{y}\left(\hat{\beta}_{GWPR}\right)}\right) - y_i + \hat{y}\left(\hat{\beta}_{GWPR}\right)\right], \tag{3.1}$$

4. The best bandwidth value is obtaining after updating the positions according to the three phases of the BWO algorithm until $t_{\max}$ is reached.

In this respect, the pseudo code of the BWO algorithm for finding the bandwidth value of the kernel, σ is given in the Algorithm 1.

4. Evaluation criteria

To compare and evaluate our proposed method, BWO-GWPR, performance with other methods, two criteria for model evaluation were used. The first criterion is the pseudo $-$ R^{2}and the second criterion is the Deviance. They are defined as, respectively,

$$\text{pseudo} - \text{R}^2 = 1 - \frac{D\left(y, \hat{y}\left(\hat{\beta}_{GWPR}\right)\right)}{D\left(y, \hat{y}\left(\hat{\beta}_0\right)\right)} \tag{4.1}$$

$$\text{Deviance} = 2 \sum_{i=1}^{n} \left[y_i \log \left(\frac{y_i}{\hat{y}\left(\hat{\beta}_{GWPR}\right)} \right) - y_i + \hat{y}\left(\hat{\beta}_{GWPR}\right) \right] \tag{4.2}$$

where $D\left(y, \hat{y}\left(\hat{\beta}_{GWPR}\right)\right)$ is the deviance of the fitted GWPR model and $D\left(y, \hat{y}\left(\hat{\beta}_0\right)\right)$ is the deviance of the Intercept-only model. The best value of the bandwidth would be the one with the highest value of pseudo $-$ R^{2}and the lowest values of the Deviance.

5. Data description

A year frame data, 2022, were collected from the 18 Iraqi provinces. The datasets for this study were obtained from Authority of Statistics and Geographic Information System, Iraq (https://cosit.gov.iq/ar/). The data included nine types of information in each individual provinces: cancer rate (average per (10000) persons), as count data, representing the response variable. Unemployment rate (X1), Urbanization rate (X2), PM2.5 (X3),NO2 (X4), SO2 (X5), O3 (X6), CO (X7), and CH4 (X8). Variables X1 to X8 represent the explanatory variables. In Figure 1, the cancer rate of 18 Iraqi provinces is reported. The geographical pattern of the cancer rate suggests differences between northern part and the southern part of the provinces.

6. Results and Discussion

First, the Kolmogorov Smirnov test was used in this study to test the goodness of fit of the response variable to the Poisson distribution. The result of the test is equal 7.486 with P-value equates 0.80128. This is pointed out by this result as an indication that Poisson distribution is a perfect fit for this response variable (cancer rate). Table 2 provides the coefficients of PR model (global model). According to the PR model, Urbanization rate (X2), PM2.5 (X3), CO (X7), and CH4 (X8) had a significant effect on cancer rate. Further, the association between cancer rate and Urbanization rate (X2), PM2.5 (X3), and CH4 (X8) was positive. That is mean if the Urbanization rate (X2), PM2.5 (X3), and CH4 (X8) increased, the probability of cancer rate increased. The association between cancer rate and CO (X7) was negative; meaning that if the CO (X7) decreased, the probability of cancer rate increased. On the other hand, Unemployment rate (X1), NO2 (X4), SO2 (X5), and O3 (X6) were not significantly associated with the cancer rate.

Second, to determine local variations in the relationship between the dependent variable and predictors for the 18 locations in the study area, the spatial heterogeneity test was conducted. The Breusch-Pagan (BP) test was used to test whether the variance of residuals is homoscedastic or heteroscedastic

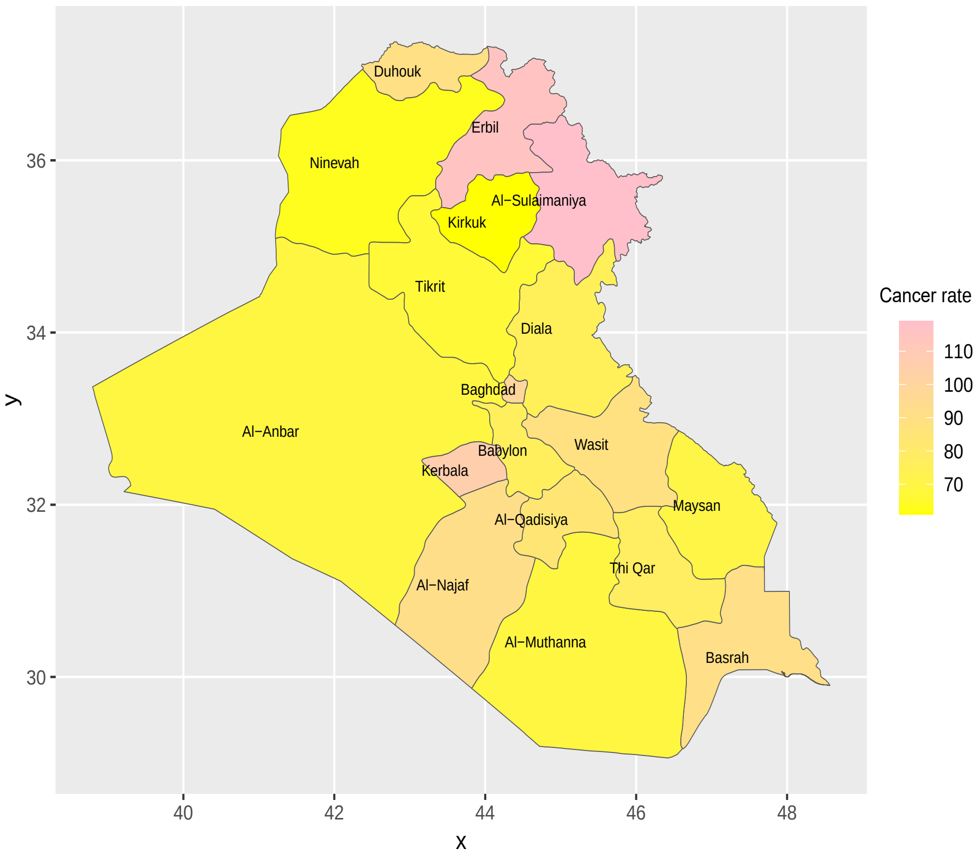

Figure 1. The spatial distribution of the cancer rate of 18 Iraqi provinces under study.

Table 2. PR model estimation

parameter	estimation	Std. error	t-value	p-value
Intercept	2.4689	0.5641	4.376	**0.0001**
X1	-0.0011	0.0083	-0.127	0.8989
X2	0.0079	0.0021	3.847	**0.0001**
X3	0.0080	0.0037	2.163	**0.0001**
X4	-0.9336	4.0006	-0.233	0.8154
X5	4.4307	3.7468	1.183	0.2370
X6	5.2732	3.8420	1.373	0.1699
X7	-0.9329	0.4526	-2.061	**0.0001**
X8	0.6388	0.2813	2.270	**0.0002**

across locations. The null hypothesis is that variances are the same in different locations while the alterative hypothesis is that there is one or more different variances in different locations. If the null hypothesis is rejected then spatial heterogeneity is said to exist significantly. As a result, the test statistic value of the BP test is 16.059 and the p-value is 0.00251 which is less than 0.05. This means that there is spatial diversity among the 18 locations in our study area. In order to model this spatial heterogeneity, the GWPR model (local models) is considered to exploring the different spatial relationships between cancer rate and the 8 explanatory variables. Further, the multicollinearity among the explanatory variables [3, 15] and the outliers [4] were checked.

Depending on the bi-square kernel weighting function, the GWPR model parameter estimates using CV, GCV, AIC, and our proposed method, BWO-GWPR, for fixed bandwidth selection are summarized in Tables 3, 4, 5, 6. The GWPR model parameters are described by the five indicators statistics: the minimum (Min), first quartile (Q1), median (Med), third quartile (Q3), and maximum values (Max). Further, Table 7 displays the results for the evaluation criteria and the bandwidth optimum value.

Two general observations are worthy of notice form Tables 3, 4, 5, 6 E1 With respect to the five statistics indicators (Min, Q1, Med, Q3, and Max), the direction (either positive or negative relationship) of relationships of relationships between cancer rate and each explanatory variable of BWO-GWPR is consistent in the corresponding counterparts in AIC, CV, and GCV methods. For example, the parameters of PM2.5 (X3) in AIC, CV, and GCV are all positive. The varying parameters of each significant variables in AIC, CV, and GCV always fall into the range of corresponding counterparts in BWO-GWPR.

Table 3. Summary of GWPR parameters for AIC method

parameter	Min	Q1	Med	Q3	Max
Intercept	1.9957	2.3020	2.4390	3.3099	3.5212
X1	-0.0001	0.0035	0.0052	0.0101	0.0105
X2	0.0067	0.0069	0.0080	0.0087	0.0089
X3	0.0061	0.0086	0.0157	0.0179	0.0181
X4	-4.4812	-4.0111	-2.4221	-2.0230	-0.8564
X5	-1.4767	-1.1745	1.0510570	5.6959049	7.1754
X6	5.3699	5.9835	8.9095	10.2804	10.5664
X7	-1.2604	-1.1007	-0.9852	-0.9464	-0.7524
X8	-0.2115	-0.0846	0.4801	0.6683	0.8184

Regarding GWPR model performance, the results in Table 7 show that both deviance and pseudo-R^2 obtained by our proposed method, BWO-GWPR, is more accurate indicated that BWO-GWPR had the highest pseudo-R^2 and least Deviance compared with AIC, CV, and GCV methods. The Deviance in the BWO-GWPR is reduced by 50.55%, 29.18%, and 18.23% respectively as compared to the AIC, CV, and GCV. The results indicate that the GWPR model using our proposed method, BWO-GWPR produces more accurate predictions for cancer rate in individual Iraqi provinces than those the AIC, CV, and GCV by capturing the spatial heterogeneity in the data.

In Figure 2, after selecting the bandwidth with BWO-GWPR, the spatial distribution of the predicted cancer rate in the GWPR model is given. In Figure 1, it is seen that the distribution of the predicted

Table 4. Summary of GWPR parameters for CV method

parameter	Min	Q1	Med	Q3	Max
Intercept	1.5176	2.2307	2.6719	3.3554	3.5354
X1	0.0017	0.0074	0.0087	0.0101	0.0105
X2	0.0064	0.0069	0.0074	0.0088	0.0092
X3	0.0042	0.0079	0.0171	0.0179	0.0183
X4	-4.6813	-4.0229	-3.6008	-2.8366	-0.2338
X5	-1.8386	-1.2337	0.4256	8.2163	10.0032
X6	5.5287	7.3340	10.0703	10.2899	10.6670
X7	-1.2929	-1.1032	-1.0252	-0.9752	-0.6934
X8	-0.2192	0.1111	0.2694	0.6708	1.0107

Table 5. Summary of GWPR parameters for GCV method

parameter	Min	Q1	Med	Q3	Max
Intercept	1.4861	2.2081	2.9839	3.5412	3.8595
X1	0.0026	0.0074	0.0087	0.0099	0.0126
X2	0.0056	0.0065	0.0071	0.0088	0.0093
X3	0.0040	0.0084	0.0174	0.0179	0.0194
X4	-6.8642	-4.5407	-3.8301	-2.6757	-0.1301
X5	-2.7364	-2.5067	-0.4729	8.2856	10.3894
X6	5.1972	7.6457	9.5011	10.2436	10.7793
X7	-1.3083	-1.0795	-0.9803	-0.9447	-0.8651
X8	-0.3666	-0.2138	0.1067	0.6904	1.0251

Table 6. Summary of GWPR parameters for BWO-GWPR method

parameter	Min	Q1	Med	Q3	Max
Intercept	1.3969	2.1933	3.0464	3.5401	4.7482
X1	0.0003	0.0070	0.0103	0.0128	0.0208
X2	0.0038	0.0053	0.0063	0.0089	0.0095
X3	0.0036	0.0093	0.0162	0.0201	0.0246
X4	-7.1083	-5.3736	-4.0113	-2.4878	0.1532
X5	-4.50450	-3.6697	-0.6533	8.2745	11.0510
X6	4.77974	6.4176	10.4754	11.2651	15.6831
X7	-1.34579	-1.1795	-1.0036	-0.9234	-0.7264
X8	-0.71630	-0.2512	-0.0127	0.7055	1.0665

Table 7. Summary of evaluation criteria and the best bandwidth for used methods

Methods	pseudo-R^2	Deviance	best bandwidth
AIC	0.8991	6.0325	18
CV	0.9289	4.2494	17
GCV	0.9390	3.6479	16
BWO-GWPR	0.9501	2.9826	15

cancer rate is quite compatible with the distribution of the real cancer rate shown in Figure 1.

The distributions of the parameters of the significant explanatory variables, Urbanization rate (X2), PM2.5 (X3), CO (X7), and CH4 (X8) over the 18 provinces of are shown in Figures 3, 4, 5, 6. The parameters show clear patterns of spatial variation. The maps indicate that the four parameter estimates using AIC, CV, GCV, and BWO-GWPR are not equal for all locations.

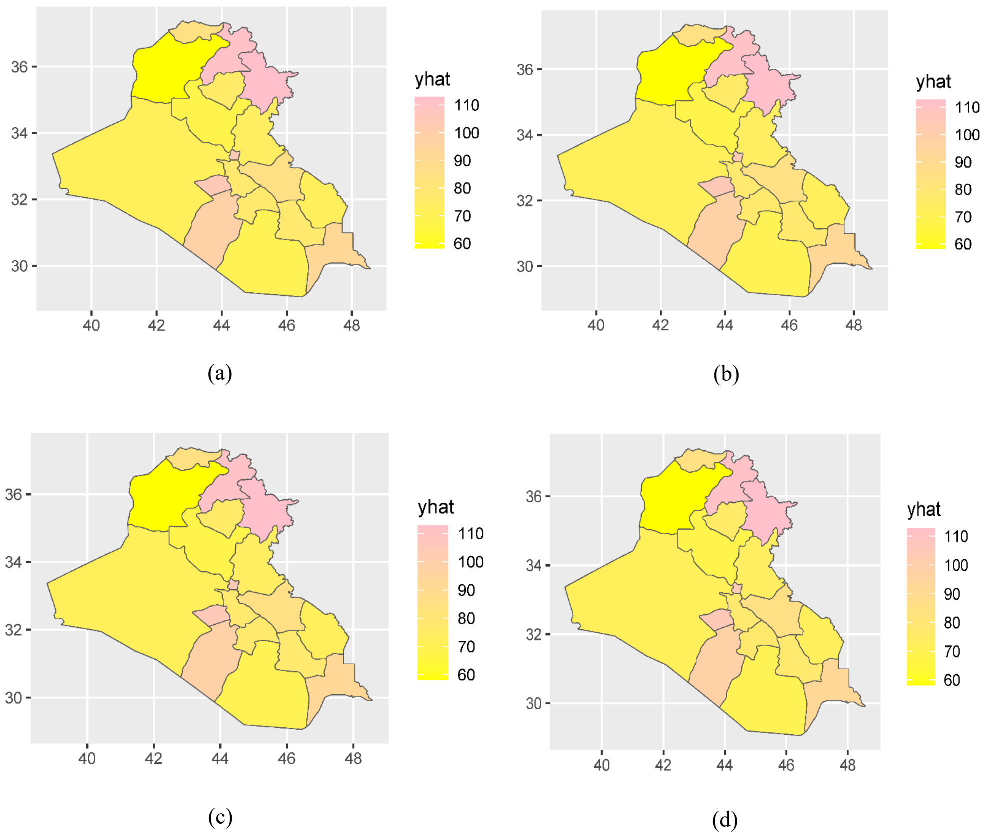

Figure 2. The cancer rate prediction (a) AIC, (b) CV, (c) GCV, and (d) BWO-GWPR

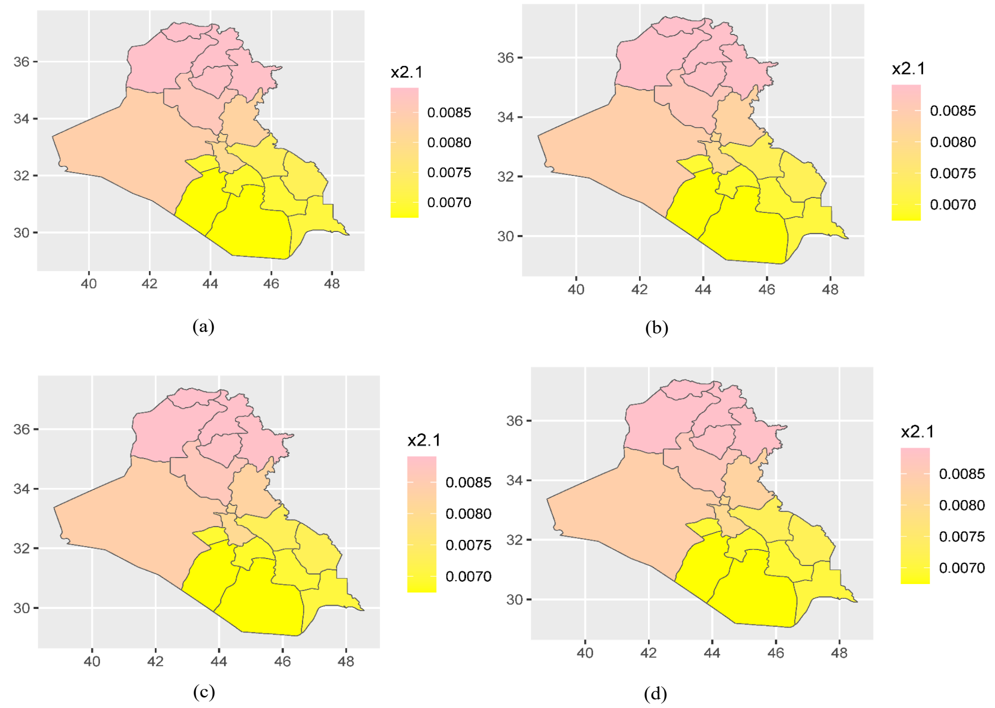

Figure 3. The significant Urbanization rate (X2) parameter estimates (a) AIC, (b) CV, (c) GCV, and (d) BWO-GWPR

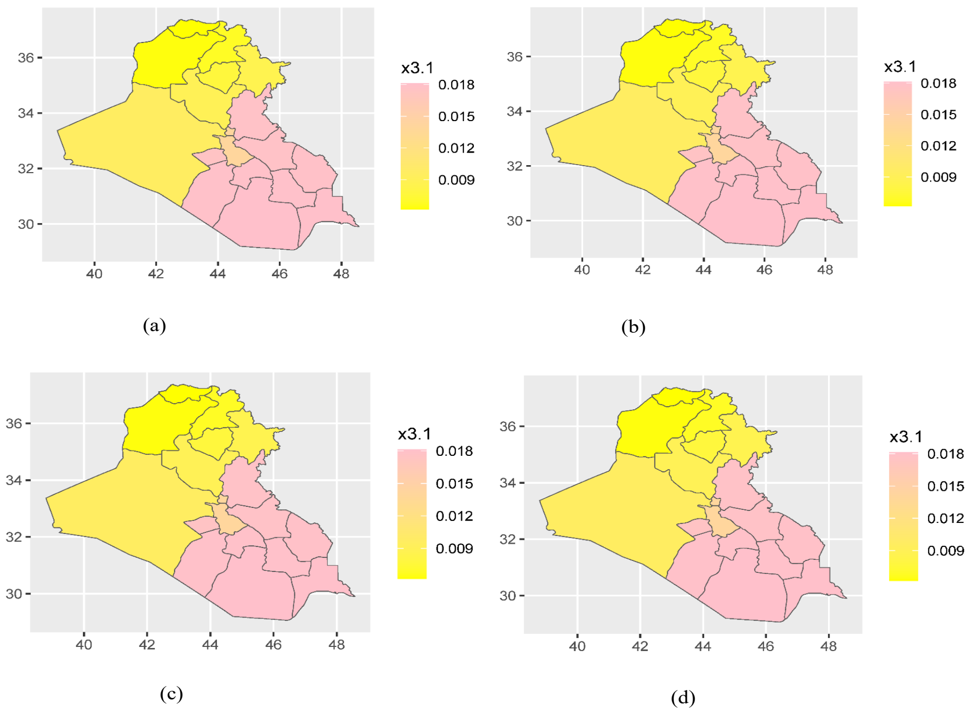

Figure 4. The significant PM2.5 (X3) parameter estimates (a) AIC, (b) CV, (c) GCV, and (d) BWO-GWPR

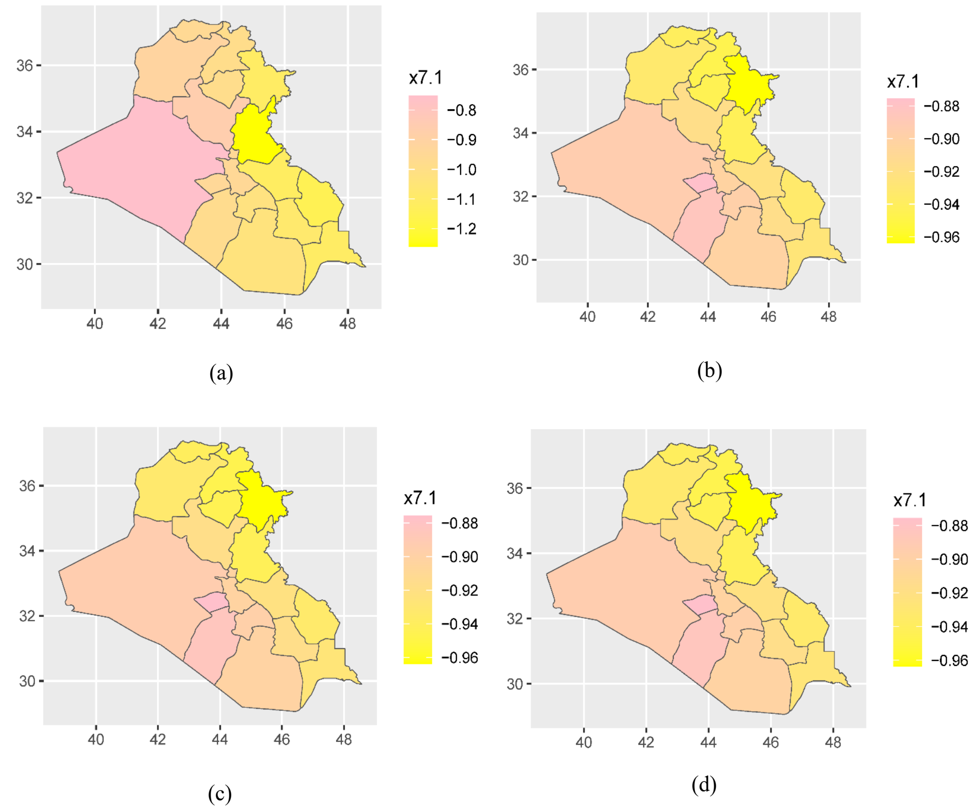

Figure 5. The significant CO (X7) parameter estimates (a) AIC, (b) CV, (c) GCV, and (d) BWO-GWPR

For further evaluation of our proposed method, a simulation study was conducted. Each model is fitted to the count data generated from the following model: $y_i \sim Poisson\left(\mu_i, \sigma^2\right)$, $\mu_i = \exp\left(\beta_{i,0} + x_{i,1}\beta_{i,1} + x_{i,2}\beta_{i,2}\right)$, $x_{i,k} \sim N(0,1)$. Where the coefficients are generated from the following spatial moving average proc-esses $\beta_{i,k} = \mu_{k(\beta)} + \sigma_{k(\beta)}\left[\frac{\sum_{i=1}^{N} g_{i,j}(b)u_{j,k}}{\sum_{i=1}^{N} g_{i,j}(b)}\right]$, where $u_{j,k} \sim N(0,1)$ *and* $[\cdot]$is an operator standardizing. with mean zero and variance one[3]. The spatial weight $g_{i,j}(r)$is given by the $(i,j) - th$ element of the spatial proximity matrix, whose $(j,j) - th$ element equals $\exp\left(-d_{i,j}^2 \big/ r^2\right)$, where r is a range parameter that determines the spatial scale of the local coefficient. Spatial coordinates for evaluating distance were generated from two uniform distributions (minimum: -2; maximum: 2). Regarding simulation study, the results in Table 8 show that both deviance and pseudo-R^2 obtained by our proposed method, BWO-GWPR, is more accurate indicated that BWO-GWPR had the highest pseudo-R^2 and least Deviance compared with AIC, CV, and GCV methods.

7. Conclusion

The GWPR model is a particular type of statistical technique applied to the analysis of count data that changes across space while taking into account while taking into account the spatial specificity of the relations between the variables. In addition, if the bandwidth is chosen poorly, the model fitting to the data set can be either overly complex and memorize the noise or insufficiently complex and fail

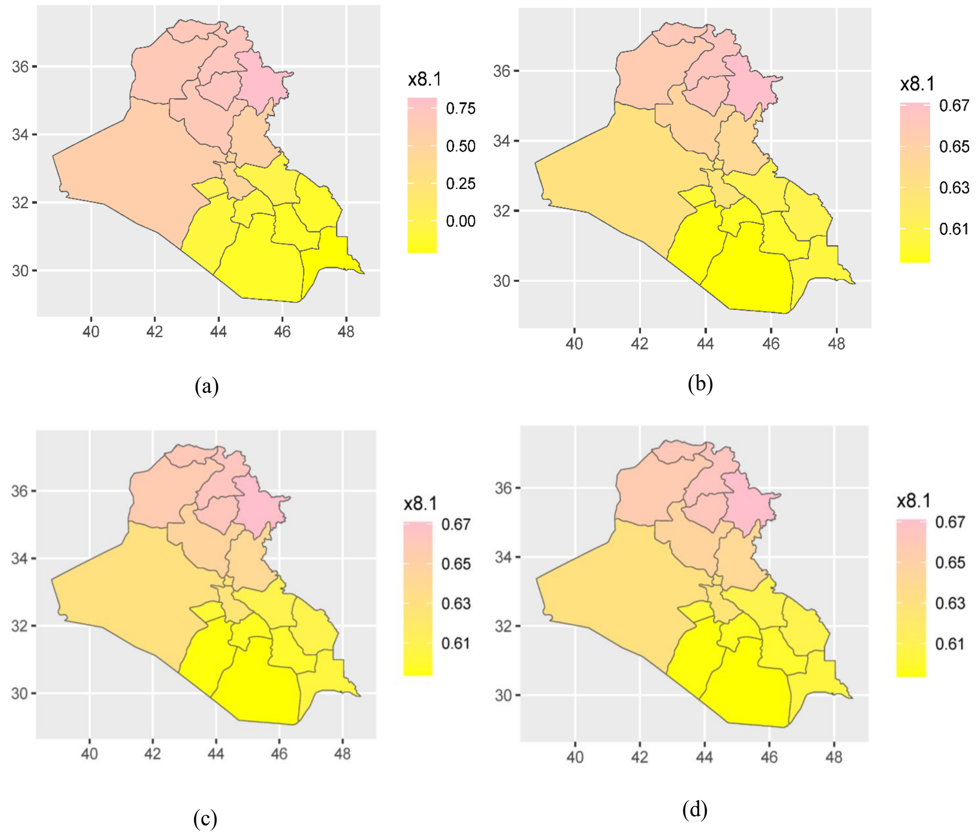

Figure 6. The significant CH4 (X8) parameter estimates (a) AIC, (b) CV, (c) GCV, and (d) BWO-GWPR

Table 8. Simulation study results

Methods	pseudo-R^2	Deviance	best bandwidth
AIC	0.8875	4.1592	22
CV	0.8981	3.8874	20
GCV	0.9055	3.6552	18
BWO-GWPR	0.9217	2.8083	13

to capture important patterns of the data set. This study presents an employing of meta-heuristic algorithms, which is beluga whale optimization algorithm for estimating the bandwidth value in GWPR model which can offer a promising alternative to traditional methods. Based on cancer rate estimation as a real data application, the comparison studies and evaluations demonstrated that the proposed method, BWO-GWPR, outperformed AIC, CV, and GCV methods regarding pseudo-R^2 and Deviance. Additionally, the varying parameters of each significant variables in AIC, CV, and GCV always fall into the range of corresponding counterparts in BWO-GWPR. However, as limitations, GWPR is prone to multicollinearity issues at local levels. In addition, GWPR has instability with zero counts. For the future work, the spatial lag model can be employed.

Data Availability: The data that support the findings of this study are available upon request from the corresponding author.
Author contributions: All authors have accepted responsibility for the entire content of this manuscript and approved its submission.
Conflicts of interest: The authors declare no conflicts of interest.

References

1. Abdel-Basset, M., Abdel-Fatah, L., & Sangaiah, A. K. (2018). Chapter 10 - Metaheuristic Algorithms: A Comprehensive Review. In A. K. Sangaiah, M. Sheng, & Z. Zhang (Eds.), Computational Intelligence for Multimedia Big Data on the Cloud with Engineering Applications (pp. 185-231): Academic Press.
2. Abdelwahab, M. M., Shalaby, O. A., Semary, H., & Abonazel, M. R. (2024). Driving factors of NOx emissions in China: Insights from spatial regression analysis. Atmosphere, 15(7), 793.
3. .Abonazel, M. R., & Dawoud, I. (2022).Developing robust ridge estimators for Poisson regression model. Concurrency and Computation: Practice and Experience, 34(15), e6979.
4. .Abonazel, M. R., & Saber, O. M. (2020). A comparative study of robust estimators for Poisson regression model with outliers. J. Stat. Appl. Probab, 9, 279-286.
5. Abonazel, M. R., Shalaby, O. A., & Youssef, A. H. (2025). New Modified Estimators for the Spatial Lag Model with Randomly Missing Data in Dependent Variable: Methods and Simulation Study. International Journal of Analysis and Applications, 23, 67-67
6. Al-Hasani, G., Asaduzzaman, M., & Soliman, A.-H. (2021). Geographically weighted Poisson regression models with different kernels: Application to road traffic accident data. Communications in Statistics: Case Studies, Data Analysis and Applications, 7(2), 166-181.doi:10.1080/23737484.2020.1869628
7. Alangood, H. N. A., Algamal, Z. Y., & Khaleel, M. A. (2024). Variable selection in Poisson regression model based on chaotic meta-heuristic search algorithm. Paper presented at the BIO Web of conferences.
8. Algamal, Z. (2019a). Variable Selection in Count Data Regression Model based on Firefly Algorithm. Statistics, Optimization & Information Computing, 7(2). doi:10.19139/soic.v7i2.566
9. Algamal, Z. (2019b). Variable selection in count data regression model based on firefly algorithm. Statistics, Optimization & Information Computing, 7(2), 520-529.

10. Algamal, Z. Y. (2012). Diagnostic in poisson regression models. Electronic Journal of Applied Statistical Analysis, 5(2), 178-186.

11. Brunsdon, C., Fotheringham, A. S., & Charlton, M. E. (1996). Geographically weighted regression: a method for exploring spatial nonstationarity. Geographical analysis, 28(4), 281-298.

12. Bui, L. V., Mor, Z., Chemtob, D., Ha, S. T., & Levine, H. (2018). Use of Geographically Weighted Poisson Regression to examine the effect of distance on Tuberculosis incidence:A case study in Nam Dinh, Vietnam. PLoS One, 13(11), e0207068. doi:10.1371/journal.pone.0207068

13. Cheng, E. M., Atkinson, P. M., & Shahani, A. K. (2011). Elucidating the spatially varying relation between cervical cancer and socio-economic conditions in England. International journal of health geographics, 10, 1-17.

14. Fotheringham, A. S., Brunsdon, C., & Charlton, M. E. (2002). Geographically Weighted Regression: The Analysis of Spatially Varying Relationships. Chichester, England: Wiley.

15. Guo, L., Ma, Z., & Zhang, L. (2008). Comparison of bandwidth selection in application of geographically weighted regression: a case study. Canadian Journal of Forest Research, 38(9), 2526-2534. doi:10.1139/x08-091

16. Hadayeghi, A., Shalaby, A. S., & Persaud, B. N. (2010). Development of planning level transportation safety tools using Geographically Weighted Poisson Regression. Accid Anal Prev, 42(2), 676-688. doi:10.1016/j.aap.2009.10.016

17. Hezaveh, A. M., Arvin, R., & Cherry, C. R. (2019). A geographically weighted regression to estimate the comprehensive cost of traffic crashes at a zonal level. Accid Anal Prev, 131, 15-24. doi:10.1016/j.aap.2019.05.028

18. Hu, X., Lu, Y., Zhang, H., Jiang, H., & Shi, Q. (2021). Selection of the bandwidth matrix in spatial varying coefficient models to detect anisotropic regression relationships. Mathematics, 9(18), 2343. doi:10.3390/math9182343

19. Koç, T. (2022). Bandwidth selection in geographically weighted regression models via information complexity criteria. Journal of Mathematics, 2022(1), 1527407. doi:10.1155/2022/1527407

20. Leong, Y. Y., & Yue, J. C. (2017). A modification to geographically weighted regression. International journal of health geographics, 16, 1-18. doi:10.1186/s12942-017-0085-9

21. Li, Z., Wang, W., Liu, P., Bigham, J. M., & Ragland, D. R. (2013). Using Geographically Weighted Poisson Regression for county-level crash modeling in California. Safety Science, 58, 89-97. doi:10.1016/j.ssci.2013.04.005

22. Ma, Y., & Gopal, S. (2018). Geographically weighted regression models in estimating median home prices in towns of Massachusetts based on an urban sustainability framework. Sustainability, 10(4), 1026. doi:10.3390/su10041026

23. Munira, S., & Sener, I. N. (2020). A geographically weighted regression model to examine the spatial variation of the socioeconomic and land-use factors associated with Strava bike activity in Austin, Texas. Journal of Transport Geography, 88, 102865. doi:10.1016/j.jtrangeo.2020.102865

24. Nakaya, T., Fotheringham, A. S., Brunsdon, C., & Charlton, M. (2005). Geographically weighted Poisson regression for disease association mapping. Stat Med, 24(17), 2695-2717. doi:10.1002/sim.2129

25. Nkeki, F. N., & Asikhia, M. O. (2019). Geographically weighted logistic regression approach to explore the spatial variability in travel behaviour and built environment interactions: Accounting simultaneously for demographic and socioeconomic characteristics. Applied Geography, 108, 47-63. doi:10.1016/j.apgeog.2019.05.008

26. Pagliara, F., & Mauriello, F. (2020). Modelling the impact of High Speed Rail on tourists with Geographically Weighted Poisson Regression. Transportation Research Part A: Policy and Practice, 132, 780-790. doi:10.1016/j.tra.2019.12.025

27. Poliart, A., Kirakoya-Samadoulougou, F., Ouédraogo, M., Collart, P., Dubourg, D., & Samadoulougou, S. (2021). Using geographically weighted Poisson regression to examine the association between socioeconomic factors and hysterectomy incidence in Wallonia, Belgium. BMC women's health, 21(1), 1-12. doi:10.1186/s12905-021-01514-y

28. Rajwar, K., Deep, K., & Das, S. (2023). An exhaustive review of the metaheuristic algorithms for search and optimization: taxonomy, applications, and open challenges. Artificial Intelligence Review, 56(11), 13187-13257. doi:10.1007/s10462-023-10470-y

29. Mathew, A., JK, K., RN, P., Sankar, A., & George, P. S. (2024). Assessment of spatial variation in lung cancer incidence and air pollutants: spatial regression modeling approach. International Journal of Environmental Health Research, 1-15. doi:10.1080/09603123.2024.2362844

30. Sisman, S., & Aydinoglu, A. C. (2022). A modelling approach with geographically weighted regression methods for determining geographic variation and influencing factors in housing price: A case in Istanbul. Land use policy, 119, 106183. doi:10.1016/j.landusepol.2022.106183

31. Soroori, E., Mohammadzadeh Moghaddam, A., & Salehi, M. (2020). Modeling spatial nonstationary and overdispersed crash data: Development and comparative analysis of global and geographically weighted regression models applied to macrolevel injury crash data. Journal of Transportation Safety & Security, 13(9), 1000-1024. doi:10.1080/19439962.2020.1712671

32. Tareke, A. A., Gashu, K. D., & Endehabtu, B. F. (2022). Geographical weighted regression analysis of delayed antenatal care initiation and its factors among all reproductive-aged women in Ethiopia, 2016. Contraception and Reproductive Medicine, 7(1), 23. doi:10.1186/s40834-022-00190-z

33. Xu, C., Wang, Y., Ding, W., & Liu, P. (2020). Modeling the Spatial Effects of Land-Use Patterns on Traffic Safety Using Geographically Weighted Poisson Regression. Networks and Spatial Economics, 20(4), 1015-1028. doi:10.1007/s11067-020-09509-2

34. Yu, H., & Peng, Z.-R. (2019). Exploring the spatial variation of ridesourcing demand and its relationship to built environment and socioeconomic factors with the geographically weighted Poisson regression. Journal of Transport Geography, 75, 147-163.

35. Zhang, H., Liu, Y., Chen, F., Mi, B., Zeng, L., & Pei, L. (2021). The effect of sociodemographic factors on COVID-19 incidence of 342 cities in China: a geographically weighted regression model analysis. BMC Infect Dis, 21(1), 428. doi:10.1186/s12879-021-06128-1

36. Zhong, C., Li, G., & Meng, Z. (2022). Beluga whale optimization: A novel nature-inspired metaheuristic algorithm. Knowledge-Based Systems, 251. doi:10.1016/j.knosys.2022.109215

37. Zhou, X., Assunção, R., Shao, H., Huang, C.-C., Janikas, M., & Asefaw, H. (2023). Gradient-based optimization for multi-scale geographically weighted regression. International Journal of Geographical Information Science, 37(10), 2101-2128. doi:10.1080/13658816.2023.2246154

38. Zhou, X., Ding, X., Yan, J., & Ji, Y. (2023). Spatial heterogeneity of urban illegal parking behavior: A geographically weighted Poisson regression approach. Journal of Transport Geography, 110, 103636.. doi:10.1016/j.jtrangeo.2023.103636