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# Enhancing Reliability and Accuracy in Stochastic Growth Modeling: Method of Three Selected Points Approach

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**Abstract:** Growth models play a pivotal role in diverse fields, such as population dynamics, epidemiology, finance, and ecological systems. Traditionally, deterministic growth models have been extensively employed to capture various aspects of growth phenomena. However, in real-world scenarios, stochasticity is inherent in the data, challenging the suitability of deterministic models. Consequently, there is a growing interest in developing stochastic growth models capable of accommodating inherent uncertainties. This study addresses three fundamental questions within the context of stochastic growth modeling. Firstly, it investigates the continued reliability of the Method of Three Selected Points (MTSP) for estimating parameters in stochastic differential equations (SDEs), given the increasing popularity of stochastic models over deterministic ones. Secondly, it explores the required form of SDEs that maximizes the success and reliability of the MTSP approach. Lastly, it conducts a comparative analysis of the MTSP method against commonly employed techniques in the literature. To address these questions, we propose a novel approach to constructing SDEs that enhance the stability of both bounded and unbounded stochastic growth models. By carefully selecting the form of the diffusion coefficient, we achieve higher accuracy in estimating the parameters of the drift coefficient. Through empirical simulations and a comprehensive reliability analysis using real stock data, we demonstrate the superior performance of the MTSP method when compared to the Pseudo-maximum likelihood method and physics-informed neural networks within the framework of SDEs. Our findings underscore the continued effectiveness of the MTSP method for estimating parameters in stochastic growth models, even in an era where stochastic models dominate. Additionally, we provide insights into the optimal structure of SDEs for maximizing the reliability of the MTSP approach. Thus, the study contributes to the ongoing dialogue surrounding stochastic growth modeling and offers a robust methodology for parameter estimation in this context, with practical applications in fields ranging from epidemiology to finance.

**Keywords:** Method of Three Selected Points · Reliability analysis · Logistic curve · Stochastic Differential Equations.
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## 1. Introduction

Growth models play a pivotal role in various domains, as they serve as essential tools for modeling the expansion of human, animal, plant, and bacterial populations. Over time, these models have found application in diverse fields, including the modeling of growth processes in plants and animals, the study of disease propagation and control such as the COVID-19 pandemic, half-life estimation, and the analysis of stock price movements. Within the literature, deterministic functions, such as sigmoid, logistic, and unbounded growth models, have been very influential and continues to be so [25, 8]. In this context, non-linear methods like the method of three selected points, with various adaptations, have been employed to estimate deterministic models. However, the stochastic nature of data in many of these applications has prompted researchers to explore stochastic versions of growth models, and these have garnered more attention compared to their deterministic counterparts.

The predominant approach to constructing stochastic growth models has traditionally involved the introduction of a linear or nonlinear Gaussian noise term into the ordinary logistic differential equation. An instance of this can be found in [21], where this approach is combined with linearization, resulting in a closed-form solution for stochastic differential equations (SDEs). The commonly employed Brownian motion process is typically stationary, although some authors have explored the utilization of fractional Brownian motion, as demonstrated in [17]. This concept is crucial in that it disrupts the typical independence of changes within a process and is particularly useful for cyclic growth patterns, notably in disease modeling. To counteract this behavior, as illustrated in [6], efforts have been made to reduce numerous ordinary differentials into a system of a few SDEs while retaining fractional integrals. This modeling approach was extended algorithmically by [12] with the maintenance of nonlinear fractional integrals.

An alternative approach involves certain authors using a set of assumptions to formulate a discrete stochastic model. They then introduce a time reduction in the model, gradually tending toward zero, thereby transforming it into a stochastic model. This methodology can be observed in the works of [2], [3], and [14]. This modeling technique is commonly favored by researchers focusing on disease modeling. For stock price growth, some authors establish mathematical relationships among the parameters of the geometric Brownian motion, effectively causing the model to behave as a growth model [5]. Another prevalent approach involves using these assumptions to derive stochastic or deterministic partial differential equations. The resulting model can then be discretized and converted into a stochastic differential equation for modeling growth, as frequently seen in [10] and [18].

In the context of available data, researchers frequently turn to methods like nonlinear and linear regression techniques, which aim to estimate parameters. Newtonian nonlinear regression, while highly accurate by considering all data points, can face challenges when relying on precise matrix universe estimation at each iteration [15]. Linear regression is effective when the growth term constant is zero, allowing for a straightforward logarithmic linearization of the model. However, for models like the reciprocal of logistic and sigmoid functions, which yield a growth function with a non-zero constant, linearization can be challenging. The Method of Three Selected (MTS) points, which employs only three data points, often leads to model underestimation, although careful selection of the three points can mitigate this issue [29].

Pseudo-likelihood algorithms, as described in [4], are commonly used for estimating the components of stochastic differential equations. These methods are relatively straightforward, but their use of independent sampling and a normal distribution for constructing likelihood may not accurately capture the underlying data characteristics. Alternatively, some researchers employ machine learning or deep learning techniques to estimate drift and diffusion coefficients. For instance, [22] combined a recurrent neural network with the Euler-Maruyama method. Others, such as [11] and [28], utilize physics-informed neural networks for parameter estimation in stochastic differential equations, while still others continue to rely on the Euler method. These approaches, although accurate, often require substantial amount of data, which may not be readily available for many growth models [24].

Hence, several crucial questions remain unanswered within the existing body of literature. The first of these inquiries pertains to the continued reliability of the method of three selected points for estimating stochastic differential equations governing growth, especially considering the rising popularity of stochastic models over their deterministic counterparts. This naturally leads to a subsequent question: What specific format should stochastic differential equations assume to ensure the method of three selected points is applied successfully and with a high degree of reliability? An extension of this query involves the method's comparative performance against the most commonly employed techniques.

To address these issues, this paper makes noteworthy contributions. We develop a novel approach to constructing stochastic differential equations, tailored to enhance the stability of both bounded and unbounded stochastic growth models. By meticulously selecting the form of the diffusion coefficient, we achieve a higher level of precision in estimating the parameters of the drift coefficient. Through the examination of stock data and an extensive reliability analysis, we demonstrate the superior performance of our approach compared to the pseudo-maximum likelihood technique and physics-informed neural networks within the context of stochastic differential equations, as presented in this study.

The subsequent sections of this paper are structured as follows: In Section 2, we delve into the derivation of stochastic differential equations and introduce a three-point method for parameter derivation in the models. Section 3 is dedicated to presenting simulations of these models to illustrate their growth characteristics. In Section 4, we thoroughly examine the results obtained from our comparative analysis, juxtaposing our method with those commonly found in existing literature. Finally, the concluding remarks and future directions are laid out in the last section.

## 2. Derivation of Stochastic Growth Model

We establish stochastic models for both unbounded and bounded growth scenarios. The ensuing lemma provides a concise summary of our derivations:

**Lemma 1.** *The stochastic model, equivalent to the deterministic model of the form $y_t = C + A(B^t)$, is defined by Equation (2.1):*

$$dy_t = k(y_t - C)dt + \sigma^2 y_t dB_t,\ y_0 = A + C. \tag{2.1}$$

*Proof.* We begin by considering the deterministic expression $y_t = C + A(B^t) = C + A\exp(kt)$, where $k = \ln(B)$. As a result, we can differentiate it with respect to time, yielding $\frac{dy_t}{dt} = Ak\exp(kt) = k(y_t - C)$. This gives the expression $dy_t = k(y_t - C)dt$.

In alignment with methodologies outlined in [20], [21] and [27], we make the assumption that the population, denoted as $y_t$ at time $t$, follows a stochastic model, represented as:

$$dy_t = k(y_t - C)dt + \xi_t dt, \tag{2.2}$$

where $\xi_t$ is the Gaussian white noise. In addition, we make the hypothesis that $\xi_t dt = y_t\sigma^2 dB_t$, where $B_t$ represents Brownian motion, and $\sigma = \sqrt{\frac{1}{N}\int_{-\infty}^{\infty}(y_t - \mathbb{E}(y_t))^2 f y_t(s)ds}$. This is based on the given probability distribution function $f_{y_t}(s)$ for $y_t$. □

This result diverges from the conventional assumption in the literature, as indicated by [7], where the model is typically formulated as: $dy_t = Ak\exp(-kx)y_t dt + \sigma y_t dB_t = k(y_t^2 - Cy_t)dt + \sigma y_t dB_t$. This implies that the drift coefficient, $k(y_t^2 - Cy_t)$, is quadratic in nature. In contrast, our model features a linear drift coefficient, $k(y_t - C)$, ensuring the uniqueness of solutions [9] within our model. Another deviation from the typical modeling approach lies in our definition of the second parameter, $\sigma$. This deviation allows us to estimate it using the following method:

$$\hat{\sigma} = \frac{1}{\sqrt{N}}\left(\frac{\sum_{i=1}^{n}(y_i - \overline{y_i})^2}{n-1}\right)^{\frac{1}{2}}. \tag{2.3}$$

In this case, $N$ is the number of discretization points for Euler-Maruyama method and $n$ is the sample size of the growth data. This approach to defining $\hat{\sigma}$ departs from the standard method. The following lemma guarantees that results in Lemma 1 can also be used for logistic growth model.

**Lemma 2.** *The model in Equation (2.1) can be used for a logistic growth model of the form:*

$$y_t = \frac{C_1}{1 + C_2 e^{\alpha t}},\ \alpha < 0. \tag{2.4}$$

*Proof.* Consider $R(t) = \frac{1}{y_t} = \frac{1+C_2e^{\alpha t}}{C_1} = \frac{1}{C_1} + \frac{C_2}{C_1}e^{\alpha t} = C + Ae^{\alpha t},\ \alpha < 0$. Thus, we can choose $C = \frac{1}{C_1},\ A = \frac{C_2}{C_1},\ B = e^{\alpha}$. □

We will now derive the stochastic model corresponding to the logistic growth model. The outcomes are presented in the following lemma:

**Lemma 3.** *The stochastic model, equivalent to the logistic growth model $y_t = \frac{C_1}{1+C_2e^{kt}}$ where $k < 0$, is described by Equation (2.5):*

$$dy_t = ky_t\left(1 - \frac{y_t}{C_1}\right)dt + \sigma dB_t,\ y_0 = \frac{C_1}{1 + C_2}. \tag{2.5}$$

*Proof.* Consider the equation $\frac{1}{y_t} = \frac{1}{C_1} + \frac{C_2}{C_1}e^{\alpha t}$. This equation can be manipulated as follows: $-y_t^{-2}\frac{dy_t}{dt} = \frac{\alpha C_2}{C_1}e^{\alpha t}$. This leads to the expression: $\frac{dy_t}{dt} = ky_t\left(1 - \frac{y_t}{C_1}\right) + \xi_t$, where $k = -\alpha > 0$.

Utilizing the same reasoning as presented for the proof in Lemma 1, we arrive at the results outlined in Equation (2.5). □

In the works of authors like [21] and [7], the typical presentation of results takes the form: $dy_t = \alpha y_t(C_1 - y_t)dt + \sigma y_t dB_t$. However, our model exhibits a slight variation in that the drift term in the literature is represented as $\alpha y_t(C_1 - y_t)$, whereas in our case, the drift coefficient is $ky_t\left(1 - \frac{y_t}{C_1}\right)$, where $k = -\alpha > 0$. This distinction sets our model apart. Likewise, the selection of $\sigma = \sqrt{\frac{1}{N}\int_{-\infty}^{\infty}(y_t - \mathbb{E}(y_t))^2 f_{y_t}(s)ds}$ represents another divergence that distinguishes our model. We also choose diffusion coefficient as $\sigma$ instead of $\sigma^2 y_t$ for increased stability. This makes the model more realistic, as it captures the fact that volatility can vary over time and in response to changes in the underlying system.

We now aim to outline the methods for three specific points that are applicable to the three lemmas under consideration. In the case of the unbounded growth model, these methods can be employed to estimate the parameters $A$, $B = e^k$, and $C$ as follows (refer to [26]):

$$\hat{B} = \sqrt[h]{\frac{y_3 - y_2}{y_2 - y_1}} = \left(\frac{y_3 - y_2}{y_2 - y_1}\right)^{\frac{1}{h}}. \tag{2.6}$$

$$\hat{A} = \frac{(y_2 - y_1)^2}{y_3 - 2y_2 + y_1}\left(\frac{y_2 - y_1}{y_3 - y_2}\right)^{\frac{x_1}{h}}. \tag{2.7}$$

$$\hat{C} = \frac{y_1 y_3 - y_2^2}{y_3 - 2y_2 + y_1}. \tag{2.8}$$

For the case of finding reciprocal of logistic model the method of three-selected points that can be derived as follows [16]:

$$C_1 = \frac{y_2^2(y_1 + y_3) - 2y_1 y_2 y_3}{y_2^2 - y_1 y_3}. \tag{2.9}$$

$$k = \frac{1}{t_2 - t_1}\ln\left(\frac{(C_1 - y_2)y_1}{(C_1 - y_1)y_2}\right). \tag{2.10}$$

$$\ln(C_2) = \ln\left(\frac{C_1 - y_1}{y_1}\right) - kt_1. \tag{2.11}$$

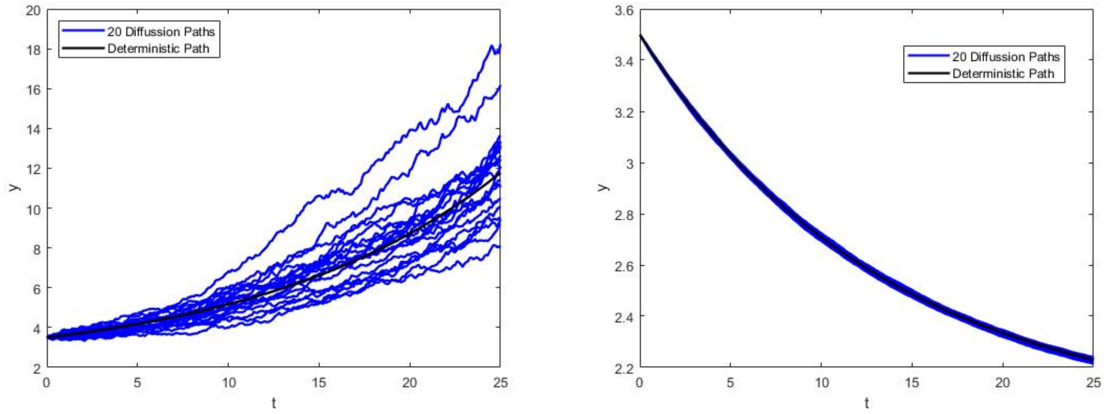
## 3. Simulation of Models

We now present the results of our simulations. We begin with the first case, which involves unbounded growth and the following parameter values: $A = 1.5$, $C = 2$, $N = n = 1000$, $k = \pm 0.075$, and a time range of $0 \le t \le 25$. To solve the model, we employ the Euler-Maruyama method [1, 13]. It is noteworthy that the stochastic model produces different growth patterns in various paths, as illustrated in Figure 1a. Furthermore, we observe that the accuracy of the model is higher when $k$ is negative, as depicted in Figure 1b. This is evident as all 20 paths closely align with the deterministic mean path. In order to conduct simulations for a stochastic model that adheres to logistic growth, we select the following parameters: $N = n = 1000$, $C_1 = 150$, $C_2 = 1.5$, $k = \pm 0.5$, and a time range of $0 \le t \le 15$. It's worth noting that the stochastic model exhibits growth behavior across different paths, as demonstrated in Figure 2a. Additionally, we observe that the accuracy remains consistent, regardless of the sign of the parameter $k$.

## 4. Comparative Analysis of Parameter Estimation Methods in Stochastic Differential Equations

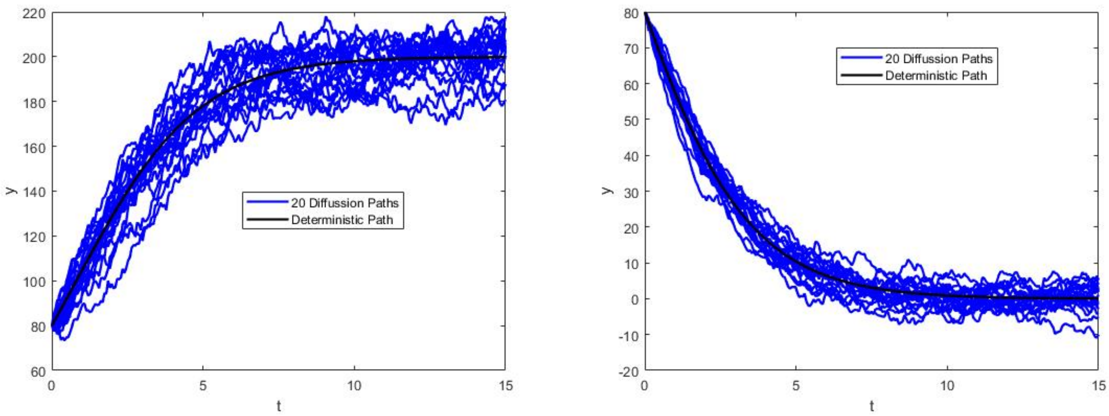
In the existing literature, two approaches are commonly employed for parameter estimation in stochastic differential equations. The first approach utilizes a physics-informed neural network (PINN), accounting for Ito's Isometry. For instance, in our models, this approach entails solving the following problem [19]:

$$\arg\min_{k,\,C} \mathcal{L} = \sum_{\forall t}\left(\frac{dy_t}{dt} - k(y_t - C)\right)^2 + \sqrt{\frac{1}{\max\{\forall t\}}\sum_{\forall t}(y_t^{model} - y_t^{data})^2}. \tag{4.1}$$




**(a)** Diffusion Paths for $k = 0.075$ **(b)** Diffusion Paths for $k = -0.075$

**Figure 1.** Simulating Unbounded Stochastic Growth Model




**(a)** Diffusion Paths for $k = 0.5$ **(b)** Diffusion Paths for $k = -0.5$

**Figure 2.** Simulating bounded Stochastic Growth Model

This results in the annihilation of diffusion term.

The second approach involves the application of a pseudo-maximum likelihood method (PMLM), assuming both the diffusion and drift coefficients are parameters of a normal distribution. For instance, in our models, this approach requires solving the following problem:

$$\arg \min_{k,\, C,\, \sigma} \mathcal{L} = \sum_{\forall t} \log(\sigma^2 y_t) + \frac{1}{2\sigma^4} \sum_{\forall t} \frac{(y_t(1-k) + kC)^2}{y_t^2} \,. \tag{4.2}$$

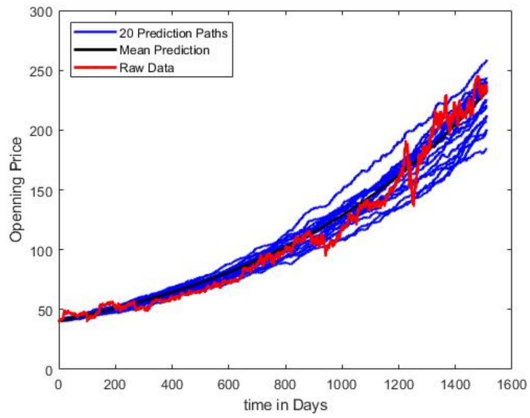
It is important to acknowledge that this assumption may not universally hold, as the solutions of stochastic differential equations (SDEs) do not always conform to a Gaussian distribution.

In this paper, we explore the suitability of a novel method based on three selected points (MTSP), aimed at minimizing the root mean square error. In practice, this involves utilizing Equation (2.3) in conjunction with equations (2.6) to (2.11) for parameter estimation. We conduct a comparative analysis of these three estimation methods, assessing their effectiveness in predicting stock data.

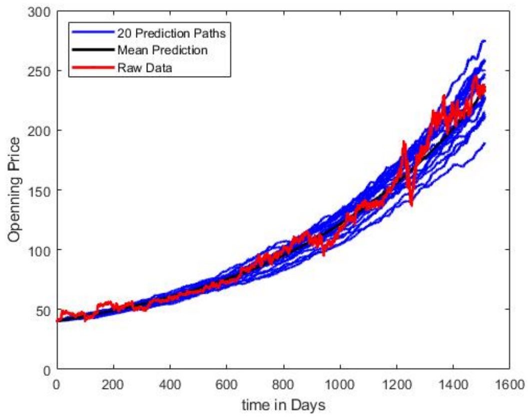
We examined Microsoft stock data, comprising daily opening prices spanning from 2015 to 2021, encompassing approximately 1,600 data points downloaded from https://www.kaggle.com/datasets/vijayvvenkitesh/microsoft-stock-time-series-analysis. Applying the Physics-Informed Neural Network (PINN), Pseudo Maximum Likelihood Method (PMLM), and the Method of Three Selected Points that minimizes mean square error, we observed that all three modeling methods yielded parameters leading to well-fitting models for the data, characterized primarily by a growth trend (see Figure 3).

The coefficients of determination were consistently high across all methods, owing to the suitability of the selected models for the given dataset. However, the method of maximum likelihood estimation (MPLM) tends to produce relatively poorer results compared to MTSP and PINN. For instance, it exhibits the lowest coefficient of determination and the highest mean absolute percentage error (Table 1). This discrepancy can be attributed to the unrealistic assumptions of independent sampling and normality distribution, which do not align with the autocorrelation and potential non-normality of data relevant to solving stochastic differential equations.

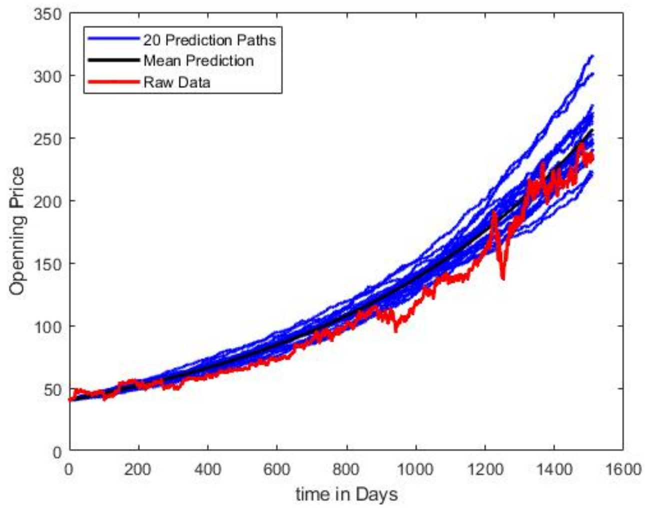
As indicated in Figure 3c, the predicted paths in MPLM tend to be consistently higher than the raw data points. While PINN displays a similar tendency, the values are much closer to the data points (Figure 3a), leading to a higher coefficient of determination and a lower mean absolute percentage error compared to MPLM, signifying greater reliability.




**(a)** Estimation through PINN is Better




**(b)** MTSP Estimates Results are Best




**(c)** Maximum Likelihood Performs Poorest

**Figure 3.** Performance through Prediction of Microsoft Stocks

Overall, MTSP demonstrates the highest reliability, with a mean absolute percentage error of 4.6% and strong suitability for growth data ($R^2$ = 98.2%, see Table 1). This explains why all paths in MTSP are the most symmetric around the mean (Figure 3b). Additionally, the choice of $\sigma$ contributed to accuracy in modeling growth, allowing us to estimate the drift parameters exclusively in PINN and MTSP, while the standard error function estimated $\sigma$. Consequently, MTSP emerges as a suitable choice for modeling stochastic growth, while PINN performs better with datasets of very large sample sizes. Therefore, it is essential to assess the dependability of these methods, which refers to the

**Table 1.** Comparison of Methods under Microsoft Stock Data

| Method | $k$ | $C$ | $\sigma$ | $R^2$ | MAPE |
|---|---|---|---|---|---|
| PINN | 0.0011 | 0.0041 | 1.9281 | 0.9767 | 0.067 |
| PMLM | 0.0012 | 0.0011 | 1.9 | 0.9455 | 0.104 |
| MTSP | 0.0014 | 13.73 | 1.909 | 0.9817 | 0.0469 |

probability that a method will result in accurate stock predictions within a mean absolute percentage error (MAPE) of 5%, assuming the study is repeated numerous times. In line with this methodology described in [23], we have calculated and provided the reliability of the methods in Table 2. This table reveals that the MTSP method demonstrates the highest reliability at 64.8% among the three methods. This confirms that the MTSP method is reliable and accurate for modeling growth within the stochastic

**Table 2.** Reliability of the Methods under Microsoft Data

| Method | PINN | PMLM | MTSP |
|---|---|---|---|
| Reliability | 0.3984 | 0.1886 | 0.6479 |

paradigm.

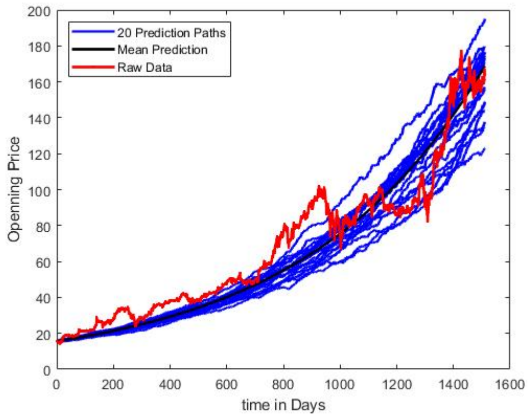
We conducted an analysis of Amazon stock data, spanning daily opening prices from 2015 to 2021, which included approximately 1,600 data points downloaded from Yahoo Finance. Notably, all three methods achieved a coefficient of determination of at least 89% (see Table 3), underscoring the suitability of the stochastic differential equation derived in this study for the provided dataset. It is worth mentioning that the Maximum Likelihood method exhibited the lowest coefficient of determination and the highest Mean Absolute Percentage Error (MAPE). However, the parameter estimates and precision measures showed minimal differences between the Probability Maximum Likelihood Method (PMLM) and Physics-Informed Neural Networks (PINN). In contrast, the Method of Three Selected Points (MTSP) outperformed the others, boasting a coefficient of determination as high as 92% (Table 3), along with the lowest MAPE, which was only 5.07%.

The major reason for the poor performance of Maximum Likelihood estimation in modeling growth using stochastic differential equations lies in its underlying assumption of normality and independent sampling

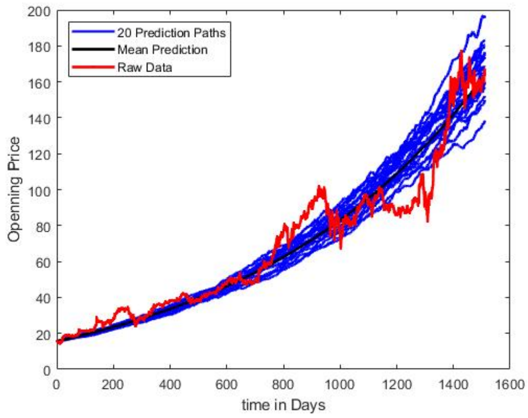
**Table 3.** Comparison of Methods under Amazon Stock Data

| Method | $k$ | $C$ | $\sigma$ | $R^2$ | MAPE |
|---|---|---|---|---|---|
| PINN | 0.0016 | 0.0023 | 1.1854 | 0.9010 | 0.1549 |
| PMLM | 0.0017 | 0.0024 | 1.1855 | 0.8999 | 0.1574 |
| MTSP | 0.0012 | -14.0676 | 1.1526 | 0.9208 | 0.0507 |

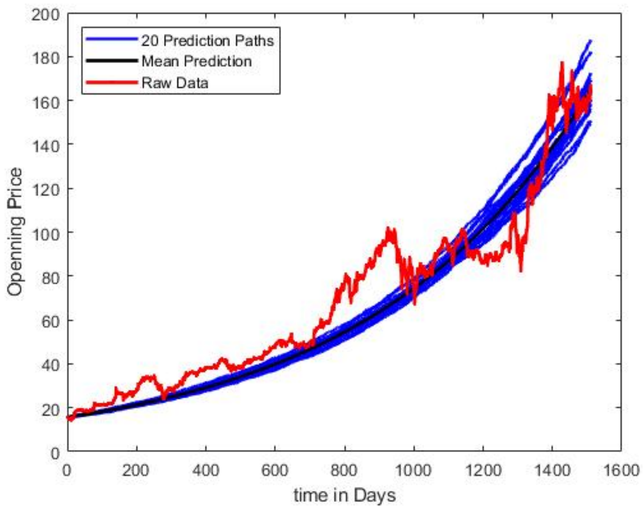
This phenomenon explains the closer proximity of the estimation lines of the MTSP model to those of the raw data (see Figure 4). The lower Mean Absolute Percentage Error (MAPE) further signifies the method's enhanced reliability. Consequently, we conducted a comprehensive reliability analysis. The results reveal that the Maximum Likelihood estimation method still maintains the highest reliability at 54% (Table 4).




**(a)** Estimation through PINN is Better




**(b)** MTSP Estimates Results are Best




**(c)** Maximum Likelihood Performs Poorest

**Figure 4.** Performance through Prediction of Amazon Stocks

**Table 4.** Reliability of the Methods under Amazon Data

| Method | PINN | PMLM | MTSP |
|---|---|---|---|
| Reliability | 0.3071 | 0.1158 | 0.5367 |

## 5. Conclusion

In this study, we have explored the intricacies of stochastic growth modeling and addressed key questions regarding the reliability and accuracy of the Method of Three Selected Points (MTSP) for parameter estimation within the context of stochastic differential equations (SDEs). Our findings offer valuable insights and contribute to the ongoing dialogue surrounding stochastic growth modeling in diverse fields. First and foremost, we have reaffirmed the enduring reliability of the MTSP method for estimating parameters in stochastic growth models. This conclusion is particularly significant in a landscape where stochastic models have gained prominence over deterministic counterparts. The MTSP approach remains a robust and dependable tool for parameter estimation, offering valuable applications in finance.

Furthermore, our investigation has shed light on the optimal structure of SDEs to maximize the success and reliability of the MTSP method. By carefully selecting the form of the diffusion coefficient, we have demonstrated how accurate parameter estimation can be achieved, even in the presence of inherent uncertainties. This insight not only enhances the applicability of the MTSP approach but also contributes to the broader understanding of stochastic growth modeling. Through comprehensive simulations and reliability analysis using real stock data, we have highlighted the superiority of the MTSP method when compared to other commonly employed techniques in the literature, such as the pseudo-maximum likelihood method and physics-informed neural networks. This empirical evidence underscores the practical advantages of the MTSP approach and its suitability for addressing the challenges posed by stochastic growth phenomena.

Finally, this study offers another perspective on stochastic growth modeling, emphasizing the continued relevance of the MTSP method in the face of evolving research trends. Future research could consider investigating appropriate parameter estimation for multivariate stochastic growth models in which growth is modeled as a function of other variables, as seen in [3] and [9]. This could be particularly intriguing given that many authors still rely on the less reliable PMLM. We have presented a robust and accurate methodology for parameter estimation within the stochastic growth paradigm. Our work opens new avenues for future research, further advancing understanding of growth processes and their modeling in the presence of uncertainty

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## Authors contributions

All the authors contributed equally at all stages of writing this article.

## Conflict of Interest

The authors declare no conflict of interest.

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