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Research article

Application of Environmental Data with New Extension of Nadarajah-Haghighi Distribution

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Abstract: The sin extension of the exponential (SExEx) or sin Nadarajah-Haghighi (SNH) distribution, a new extension of the exponential distribution with two parameters, is introduced and studied in this paper. For the SNH distribution, the hazard, survival functions and some sumarized data are explored. Methods for estimating the SNH distribution parameters using maximum likelihood estimation (MLE) and maximum product spacing (MPS) are explored. To compare various estimating techniques, a numerical analysis using Monte-Carlo simulation and real data analysis is carried out. Environmental data show how the new model is superior to some well-known distributions. In comparison to several well-known distributions, the SNH model can give better matches.

Keywords: Sin-G family; Exponential distribution; Maximum likelihood; Maximum Product Spacing; environmental data
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1. Introduction

Numerous scholars have provided examples of methods for introducing probability models. A more robust family of distributions is created by the phenomenon of adding parameters, and these distributions are successfully applied to the modeling of data sets in the fields of engineering, economics, biological research, and environmental sciences.

A different method of constructing new life distributions by altering trigonometric functions to produce new statistical distributions was provided. By modifying trigonometric functions to produce new statistical distributions, Kumar et al. [1] showed yet another method of creating a new life distribution. They created the sine-G class, a novel statistical distribution based on the sine function, which the cumulative distribution function (CDF) and the probability density function (PDF) are available as follows

$$F(y) = \sin\left(\frac{\pi}{2}G(y)\right), \quad y \in R, \tag{1.1}$$

and

$$f(y) = \frac{\pi}{2}g(y)\cos\left(\frac{\pi}{2}G(y)\right), \quad y \in R. \tag{1.2}$$

The associated hazard rate function (h) and reversed hazard rate function (rh) of S-G family are provided by

$$h(y) = \frac{\pi}{2}g(y)\tan\left[\frac{\pi}{4}(1 + G(y))\right], \tag{1.3}$$

and

$$rh(y) = \frac{\pi}{2}g(y)\cot\left(\frac{\pi}{2}G(y)\right). \tag{1.4}$$

Al-Babtain et al. [2] obtained Sine Topp-Leone-G family of distributions. Souza et al. [3] discussed Sin-G class of distributions with theory, model and application. Jamal et al. [4] introduced sine extended odd Fréchet-G family. Mahmood et al. [5] derived a new sine-G family of distributions. Tomy and Chesneau [6] obtained sine modified Lindley distribution. Alyami et al. [7] discussed modeling to factor productivity of the united kingdom food chain by using sine-exponentiated Weibull-H (SEW-H) family. Fayomi et al. [8] discussed sine inverse Lomax generated family. Aldahlan [9] introduced sine Fréchet model with application of COVID-19 death cases in kingdom of Saudi Arabia. Ahmadini [10] introduced statistical inference of sine inverse Rayleigh distribution.

Nadarajah and Haghighi [11] introduced the Nadarajah-Haghighi (NH) or extension of the exponential (ExEx) distribution. Following are the ExEx distribution's CDF and PDF, respectively:

$$G(y; \alpha, \lambda) = 1 - e^{1-(1+\lambda y)^{\alpha}}; \quad y > 0, \alpha, \lambda > 0, \tag{1.5}$$

$$g(y; \alpha, \lambda) = \alpha\lambda(1 + \lambda y)^{\alpha-1}e^{1-(1+\lambda y)^{\alpha}}; \quad y > 0, \alpha, \lambda > 0. \tag{1.6}$$

The associated hr and rhr of NH distribution are provided by

$$h(y; \alpha, \lambda) = \alpha\lambda(1 + \lambda y)^{\alpha-1}; \quad y > 0.\alpha, \lambda > 0. \tag{1.7}$$

and

$$rh(y; \alpha, \lambda) = \frac{\alpha\lambda(1 + \lambda y)^{\alpha-1}e^{1-(1+\lambda y)^{\alpha}}}{1 - e^{1-(1+\lambda y)^{\alpha}}} \tag{1.8}$$

The ExEx or NH distribution is used as a baseline model in many papers and applications as: Modified and extended versions of the ExEx distribution have been studied by many authors as Gómez et al. [12] presented a new extension of the exponential distribution based on mixtures of positive distributions. Khan et al. [29] studied transmuted generalized exponential distribution for analyzing lifetime data. Muhammad [14] introduced a new family of distributions called the Poisson-odd generalized exponential distribution. De Andrade et al. [15] introduced exponentiated generalized extended exponential distribution. Kumar et al. [16] discussed extended exponential distribution based on order statistics. Hassan et al. [17] proposed a new distribution called the alpha power transformed extended exponential distribution. Pena-Ramirez et al. [18] proposed a new lifetime model called the exponentiated power generalized Weibull distribution, which is obtained from the exponentiated family applied to the power generalized Weibull distribution.

This essay seeks to make two points clear. First, suggest and research a brand-new lifespan distribution based on the sin-G family called the sin extension of the exponential (SExEx) or sin Nadarajah-Haghighi (SNH) distribution. The SNH distribution's statistical characteristics are provided. Second, the MLE and MPS methods for parameter estimation for the SNH distribution are explored. The performance of the estimators is evaluated through a thorough simulation exercise. Environmental data are used to illustrate our SNH model as well as a few other well-known distributions. Compared to several other distributions, the SNH distribution can offer better fits.

The paper is organized as follows: Section 2 of the study introduces the description, notation, and statistical characteristics of the SNH distribution. The parameter estimation of the SNH distribution is covered in section 3. In section 4, Monte-Carlo simulation studies that assess the effectiveness of the parameter estimate for various approaches are described. Environmental data application is examined in section 5. Finally, we address the findings and conclusions of the present study in section 6.

2. Sin-NH Distribution

The Sin family and NH distribution have been used to generate SNH distribution. It is represented by the random variable $Y \sim SNH(\alpha, \lambda)$. By using Equations (1.1, 1.5, 1.2 and 1.6), the CDF and PDF of SNH distribution takes this form respectively:

$$F(y;\alpha,\lambda) = \sin\left[\frac{\pi}{2}\left(1 - e^{1-(1+\lambda y)^{\alpha}}\right)\right]; \;\; y > 0, \alpha, \lambda > 0, \tag{2.1}$$

and

$$f(y;\alpha,\lambda) = \frac{\pi}{2}\alpha\lambda(1+\lambda y)^{\alpha-1}e^{1-(1+\lambda y)^{\alpha}}\cos\left[\frac{\pi}{2}\left(1 - e^{1-(1+\lambda y)^{\alpha}}\right)\right]; \;\; y > 0, \alpha, \lambda > 0. \tag{2.2}$$

The associated survival, hazard rate (hr) and reversed hazard rate (rhr) of SNH distribution are provided by

$$S(y;\alpha,\lambda) = 1 - \sin\left[\frac{\pi}{2}\left(1 - e^{1-(1+\lambda y)^{\alpha}}\right)\right]; \;\; y > 0, \alpha, \lambda > 0, \tag{2.3}$$

$$hr(y;\alpha,\lambda) = \frac{\pi}{2}\alpha\lambda(1+\lambda y)^{\alpha-1}e^{1-(1+\lambda y)^{\alpha}}\tan\left[\frac{\pi}{4}\left(2 - e^{1-(1+\lambda y)^{\alpha}}\right)\right] \tag{2.4}$$

and

$$rhr(y;\alpha,\lambda) = \frac{\pi}{2}\alpha\lambda(1+\lambda y)^{\alpha-1}e^{1-(1+\lambda y)^{\alpha}}\cot\left[\frac{\pi}{2}\left(1 - e^{1-(1+\lambda y)^{\alpha}}\right)\right] \tag{2.5}$$

In Figure 1 shows plots of the PDF and hazard rate of the SNH distribution with some values of parameters.

If Y is a continuous variable, then FX has range frome 0 to 1 is the expression for its SNH cumulative distribution function. From this description, it follows that a percentile function Q typically returns a threshold value y below which a random sample from the provided cdf would fall q percent of the time. By using numerical analysis, we can obtain the Summarized data of SNH distribution as discussed in Table 1, where this table obtained Minimum, first quintile (Q1), Median, third quintile (Q3), maximum, and standard deviation (SD).

Table 1. Summarized data of SNH distribution

α	λ	Minimum	Q1	Median	Mean	Q3	Maximum	SD
0.5	0.02	0.0156	22.5466	49.9098	85.2389	108.6612	915.0369	106.4600
	0.52	0.0154	0.7842	1.9197	3.3587	4.2825	26.8458	4.0943
	1.02	0.0012	0.3289	0.8960	1.5919	2.0311	23.4913	2.1502
	1.52	0.0047	0.2533	0.6365	1.0957	1.3714	9.9460	1.3056
	2.02	0.0003	0.1926	0.4532	0.8531	1.0072	9.8786	1.1488
	2.52	0.0001	0.1543	0.3844	0.6999	0.8605	11.3216	1.0080
	3.02	0.0002	0.1101	0.2706	0.5182	0.6398	5.4311	0.6624
	3.52	0.0001	0.1030	0.2492	0.4732	0.6007	6.3322	0.6201
	4.02	0.0002	0.1014	0.2484	0.3994	0.5125	5.9096	0.4974
	4.52	0.0002	0.0824	0.2077	0.3529	0.4485	4.1520	0.4297
1.5	0.02	0.0052	6.6047	12.9771	16.6195	23.4745	84.1203	13.9047
	0.52	0.0051	0.2322	0.4992	0.6460	0.9187	2.8154	0.5540
	1.02	0.0004	0.0992	0.2369	0.3098	0.4447	1.8848	0.2752
	1.52	0.0016	0.0754	0.1665	0.2152	0.2998	1.0040	0.1813
	2.02	0.0001	0.0573	0.1198	0.1630	0.2217	0.8698	0.1438
	2.52	0.0000	0.0459	0.1005	0.1320	0.1860	0.8297	0.1179
	3.02	0.0001	0.0332	0.0729	0.1020	0.1428	0.5270	0.0913
	3.52	0.0000	0.0309	0.0663	0.0913	0.1308	0.5272	0.0813
	4.02	0.0001	0.0300	0.0646	0.0797	0.1124	0.4762	0.0658
	4.52	0.0001	0.0246	0.0547	0.0702	0.0988	0.3770	0.0594
3	0.02	0.0026	3.1999	6.1146	7.4308	10.6113	31.8903	5.7190
	0.52	0.0026	0.1128	0.2352	0.2879	0.4146	1.0956	0.2284
	1.02	0.0002	0.0484	0.1120	0.1385	0.2017	0.6956	0.1134
	1.52	0.0008	0.0367	0.0786	0.0963	0.1358	0.3877	0.0752
	2.02	0.0001	0.0278	0.0566	0.0727	0.1006	0.3269	0.0586
	2.52	0.0000	0.0223	0.0474	0.0588	0.0841	0.3008	0.0479
	3.02	0.0000	0.0162	0.0347	0.0457	0.0650	0.2019	0.0379
	3.52	0.0000	0.0150	0.0314	0.0408	0.0592	0.1960	0.0335
	4.02	0.0001	0.0146	0.0304	0.0358	0.0510	0.1759	0.0272
	4.52	0.0001	0.0120	0.0258	0.0315	0.0448	0.1426	0.0247

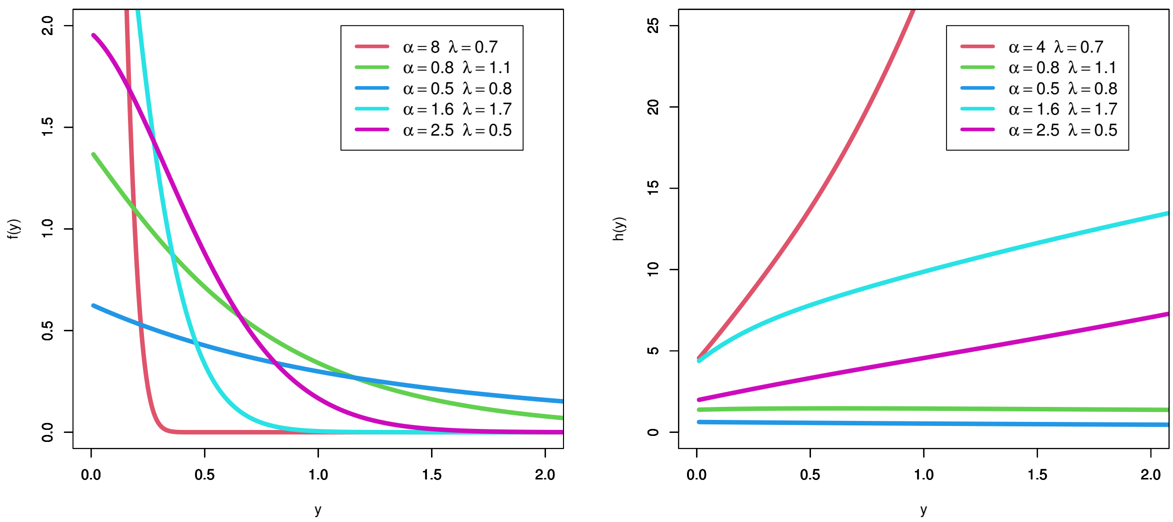

Figure 1. Plots of the PDF and hazard rate of the SNH distribution with Some Values of Parameters

3. Estimation Methods

The parameter estimate for the SNH distribution using the MLE and MPS methods will be covered in detail in this section.

3.1. Maximum likelihood estimation

The log-likelihood function of SNH distribution, is given by:

$$\begin{aligned}\ell(\alpha,\lambda) =& n\left[\log\left(\frac{\pi}{2}\right)+\log(\alpha)+\log(\lambda)\right]+(\alpha-1)\sum_{i=1}^{n}\log\left(1+\lambda y_i\right)+n-\sum_{i=1}^{n}(1+\lambda y_i)^{\alpha}+\\ &\sum_{i=1}^{n}\log\left(\cos\left[\frac{\pi}{2}\left(1-e^{1-(1+\lambda y_i)^{\alpha}}\right)\right]\right),\end{aligned} \tag{3.1}$$

To achieve the desired MLE, we will evaluate MLEs of α, and λ.

The likelihood equations are constructed with respect to the variable of interest by calculating the derivatives of Equation (3.1) in the following forms

$$\begin{aligned}\frac{\partial\ell(\alpha,\lambda)}{\partial\alpha} =& \frac{n}{\alpha}+\sum_{i=1}^{n}\log\left(1+\lambda y_i\right)-\sum_{i=1}^{n}(1+\lambda y_i)^{\alpha}\log\left(1+\lambda y_i\right)+\\ &\frac{\pi}{2}\sum_{i=1}^{n}\log(1+\lambda y_i)(1+\lambda y_i)^{\alpha}e^{1-(1+\lambda y_i)^{\alpha}}\tan\left[\frac{\pi}{2}\left(1-e^{1-(1+\lambda y_i)^{\alpha}}\right)\right],\end{aligned} \tag{3.2}$$

$$\frac{\partial \ell(\alpha,\lambda)}{\partial \lambda} = \frac{n}{\lambda} + (\alpha-1)\sum_{i=1}^{n}\frac{y_i}{1+\lambda y_i} - \alpha\sum_{i=1}^{n} y_i\,(1+\lambda y_i)^{\alpha-1} + \frac{\pi}{2}\alpha\sum_{i=1}^{n} y_i(1+\lambda y_i)^{\alpha-1} e^{1-(1+\lambda y_i)^{\alpha}} \tan\left[\frac{\pi}{2}\left(1-e^{1-(1+\lambda y_i)^{\alpha}}\right)\right], \tag{3.3}$$

The ML estimators (MLEs) of the SNH parameters can be obtained by simultaneously solving the two Equations (3.2)–(3.3) and equating them to zero. It is obvious that the MLEs cannot be computed in closed forms, but they may be quantitatively estimated using appropriate iterative methods like the Newton-Raphson. The observed information matrix, $I_{ij}(\alpha,\lambda)$, is needed in order to create the confidence intervals (CIs) of the model parameters, and it has the following form:

$$I_{ij}(\alpha,\lambda) = \begin{bmatrix} E\left[-\frac{\partial^2\ell(\alpha,\lambda)}{\partial\alpha^2}\right] & E\left[-\frac{\partial^2\ell(\alpha,\lambda)}{\partial\alpha\partial\lambda}\right] \\ E\left[-\frac{\partial^2\ell(\alpha,\lambda)}{\partial\alpha\partial\lambda}\right] & E\left[-\frac{\partial^2\ell(\alpha,\lambda)}{\partial\lambda^2}\right] \end{bmatrix} \tag{3.4}$$

Practically, the approximate asymptotic variance-covariance matrix, $V(\alpha,\lambda) = I^{-1}(\alpha,\lambda)$ is obtained by eliminating the expectation operator given in (3.4) and substituting $\hat{\alpha}$, and $\hat{\lambda}$ by their MLEs. Hence, for central limiting theory (large samples), $100(1-\delta)\%$ C Is for the model parameters α and β are

$$\hat{\alpha} \mp Z_{\delta/2}\sqrt{V(\hat{\alpha})}, \quad \hat{\lambda} \mp Z_{\delta/2}\sqrt{V(\hat{\lambda})}$$

3.2. Maximum Product Spacing

The MPS approach was developed by Cheng and Amin [20] as a substitute for the MLE method for estimating the parameters of continuous univariate distributions. They claimed that the MPS technique possesses the majority of the maximum likelihood qualities and that the likelihood function was replaced by the product of spacings. The MPS technique was also independently proposed by Ranneby [21] as a way to approximate the Kullback-Leibler measure of information.

The authors also noted that the MPSEs are at least as effective as the MLEs when they depart. The consistency and asymptotic features of the MPSEs are explored in Cheng and Amin [20]. The invariance property of MPSEs was studied by Coolen and Newby [22] and they claimed that it is identical to that of MLEs. Additionally, the MPSEs are quite efficient, and many authors suggested using them as a good substitute for the MLEs. They also discovered that in a number of circumstances, both in complete and censored samples, this estimation approach can produce better estimates than the maximum likelihood approach. The reader can consult Ghosh and Jammalamadaka [23], Rahman and Pearson [24], Singh et al. [1], Basu et al. [25], Almetwally and Almongy [12], Alshenawy et al. [26], Almetwally et al. [27], and El-Sherpieny et al. [28] for further information.

Consider sorted sample, say $Y_{1:n} < \cdots < Y_{n:n}$, from the SNH distribution with with CDF (2.1) and parameters α, and λ. Then, the uniform spacings of this random sample are defined as

$$D_i\left(y_{i:n};\alpha,\lambda\right) = \begin{cases} F\left(y_{1:n};\alpha,\lambda\right) & \text{if } i=1, \\ F\left(y_{i:n};\alpha,\lambda\right) - F\left(y_{i-1:n};\alpha,\lambda\right) & \text{if } i=2,\cdots,n, \\ 1-F\left(y_{n:n};\alpha,\lambda\right) & \text{if } i=n, \end{cases} \tag{3.5}$$

The MPSEs can be obtained by maximizing the product of spacings (3.5).

$$PS\left(y_{i:n};\alpha,\lambda\right)=C\prod_{i=1}^{n}D(y_{i:n},\alpha,\lambda)\ ,$$

where C is a constant term and it does not depend on parameters α, and λ.

Using the CDF of the SNH distribution and the logarithm of product of spacings, we obtain

$$\begin{aligned}S\left(y_{i:n};\alpha,\lambda\right)=&\log\left(\sin\left[\frac{\pi}{2}\left(1-e^{1-(1+\lambda y_{1:n})^{\alpha}}\right)\right]\right)\log\left(1-\sin\left[\frac{\pi}{2}\left(1-e^{1-(1+\lambda y_{n:n})^{\alpha}}\right)\right]\right)\\&\sum_{i=2}^{m}\log\left(\sin\left[\frac{\pi}{2}\left(1-e^{1-(1+\lambda y_{i:n})^{\alpha}}\right)\sin\left[\frac{\pi}{2}\left(1-e^{1-(1+\lambda y_{i-1:n})^{\alpha}}\right)\right]\right]\right),\end{aligned}\tag{3.6}$$

By resolving nonlinear equations and differentiating the logarithm of the product of spacing in Equation (3.6) with respect to each parameter, the MPSEs of α, and λ are produced. Nonlinear optimization procedures like the Newton-Raphson method can be used to solve problems analytically. Additionally, the ACI is followed by a normal approximation confidence interval and an asymptotic variance-covariance matrix.

4. Simulation

In order to compare the estimators of parameters of the SNH distribution, a simulation study was performed utilizing 1000 samples for each simulation for different actual values of parameters. To generate samples from the SNH distribution, with initial values $\alpha = 0.5,\ 2$ and $\lambda = 0.5,\ 2$, and 4 in Table 2, and $\alpha = 4$ and $\lambda = 0.02,\ 0.5,\ 2$, and 4 in Table 3. The sample sizes have been changed in simulation study as 30, 75, and 150. The bias, mean square error (MSE), and length of ACI of the various methods used by the estimators of the α and λ outcomes have been compared (LACI).

Table 2. MLE and MPS for different measure: when α =0.5 and 2

				MLE			MPS		
α	λ	m		Bias	MSE	LACI	Bias	MSE	LACI
0.5	0.5	30	α	0.1340	0.0910	1.0603	-0.0089	0.0380	0.9065
			λ	0.0111	0.1128	1.3167	0.2051	0.2109	1.4174
		75	α	0.0538	0.0262	0.5988	-0.0143	0.0155	0.5533
			λ	0.0027	0.0426	0.8091	0.1043	0.0675	0.8321
		150	α	0.0245	0.0105	0.3912	-0.0140	0.0075	0.3727
			λ	0.0019	0.0226	0.5889	0.0600	0.0300	0.5793
	2	30	α	0.1550	0.1896	1.5961	0.0085	0.0304	0.8298
			λ	-0.1937	0.6666	3.1108	0.1573	0.4343	2.1590
		75	α	0.0728	0.0743	1.0299	-0.0054	0.0083	0.4355
			λ	-0.1159	0.3508	2.2781	0.1054	0.1882	1.4290
		150	α	0.0179	0.0052	0.2742	-0.0087	0.0028	0.2295
			λ	-0.0395	0.1458	1.4896	0.0690	0.0963	1.0782
	4	30	α	0.0508	0.0354	0.7100	-0.0122	0.0082	0.4156
			λ	-0.1570	0.6642	3.1366	0.1236	0.2436	1.5937
		75	α	0.0293	0.0087	0.3481	-0.0100	0.0020	0.2097
			λ	-0.1262	0.6029	3.0049	0.0825	0.0897	0.9205
		150	α	0.0057	0.0014	0.1435	-0.0089	0.0010	0.1307
			λ	-0.0318	0.1088	1.2875	0.0527	0.0460	0.7312
2	0.5	30	α	0.0454	0.3927	2.4511	-0.3598	0.4739	2.7069
			λ	0.1553	0.2779	1.9757	0.3530	0.4804	2.1415
		75	α	0.0080	0.2775	2.0659	-0.2382	0.2571	2.0019
			λ	0.0828	0.0905	1.1347	0.1697	0.1427	1.2368
		150	α	-0.0174	0.1042	1.2639	-0.1538	0.1310	1.4213
			λ	0.0366	0.0313	0.6784	0.0811	0.0432	0.7057
	2	30	α	0.4908	1.4970	4.3956	-0.1539	0.3086	2.7381
			λ	0.0493	1.6271	4.9990	0.4110	0.8846	2.9563
		75	α	0.3368	0.8719	3.4155	-0.1176	0.1720	2.0142
			λ	0.0740	1.0513	4.0108	0.2533	0.3845	2.0407
		150	α	0.1732	0.4090	2.4144	-0.0968	0.1029	1.4697
			λ	0.0251	0.4696	2.6859	0.1680	0.1981	1.4736
	4	30	α	0.7309	2.0135	4.7702	-0.1177	0.1722	2.4091
			λ	-0.2207	2.3977	6.0110	0.3127	0.6863	2.4754
		75	α	0.3269	0.9227	3.5424	-0.0870	0.0885	1.5300
			λ	-0.0401	1.5450	4.8724	0.1991	0.3065	1.7869
		150	α	0.3485	0.8133	3.2620	-0.0832	0.0808	1.4979
			λ	-0.0733	1.4414	4.6999	0.1798	0.2768	1.6859

Table 3. MLE and MPS for different measure: when α =4

			MLE			MPS		
λ	m		Bias	MSE	LACI	Bias	MSE	LACI
0.02	30	α	-5.795E-05	3.510E-06	7.344E-03	-9.464E-04	8.574E-04	1.157E-01
		λ	6.782E-04	8.441E-06	1.108E-02	-3.967E-04	7.463E-06	1.168E-02
	75	α	1.689E-06	1.038E-10	3.941E-05	-8.777E-05	7.290E-06	1.067E-02
		λ	2.939E-04	3.183E-06	6.902E-03	-2.509E-04	3.000E-06	7.266E-03
	150	α	2.638E-07	4.623E-11	2.665E-05	-2.104E-06	4.655E-11	2.782E-05
		λ	5.973E-05	1.385E-06	4.609E-03	-2.598E-04	1.407E-06	4.858E-03
0.5	30	α	0.0156	0.1351	1.4401	-0.0813	0.0461	0.8766
		λ	0.0310	0.0442	0.8156	0.0038	0.0071	0.3581
	75	α	0.0296	0.1330	1.4618	-0.0432	0.0146	0.5159
		λ	0.0089	0.0054	0.2866	0.0002	0.0022	0.1918
	150	α	0.0075	0.0142	0.4666	-0.0307	0.0075	0.3555
		λ	0.0011	0.0012	0.1357	-0.0021	0.0010	0.1240
2	30	α	0.2155	1.9499	5.4109	-0.1932	0.2190	2.0803
		λ	0.3840	1.9921	5.3267	0.0936	0.1255	1.6303
	75	α	0.0908	0.7432	3.3623	-0.1236	0.0975	1.3392
		λ	0.1180	0.4509	2.5926	0.0539	0.0466	0.8839
	150	α	0.0816	0.5803	3.1492	-0.1158	0.0899	1.3023
		λ	0.1016	0.4082	2.2683	0.0453	0.0408	0.8427
4	30	α	0.4886	2.5148	5.9169	-0.1602	0.1637	2.1057
		λ	0.2172	2.9808	6.7175	0.0824	0.1376	1.5530
	75	α	0.1760	1.4708	4.7061	-0.1030	0.0731	1.2595
		λ	0.2282	1.7815	5.1577	0.0517	0.0562	1.0838
	150	α	0.1801	1.4073	4.5986	-0.0985	0.0677	1.2235
		λ	0.1713	1.6242	4.9531	0.0428	0.0494	0.9839

Tables 2, and 3 display the findings of the bias of estimate parameters and their MSE, and also the results of the ACL of the 95 percent confidence intervals. The following conclusions can be made based on the findings:

- From tables, it can be seen that as sample size rises, bias, MSEs and LACI decreases.
- MPSEs have the lowest MSEs for parameters in some times.
- The ACI for MPS provides more accurate results than the ACI for MLE, as shown in Tables 2, and 3 for various sample sizes.
- When parameter λ increases, the bias, MSE and LACI increases for parameters of SNH distribution.
- When parameter α increases, the bias, MSE and LACI increases for parameters of SNH distribution.

5. Application of real data

Line-transect distance sampling typically uses observed target distances from transect lines to estimate population densities in order to simulate detectability. The current situation is linked to significant wild animal populations in a specific setting. All creatures can be found where they initially appear, which is the basic tenet of this approach. Animal migration that is not controlled by the transect and observer could therefore significantly disturb the community's natural food chain. This data set, taken from Patil et al. [19], shows the locations of the 68 stakes found while walking L = 1000 m and looking w = 20 m on either side of the transect line. The dimensions are:

2.0, 0.5, 10.4, 3.6, 0.9, 1.0, 3.4, 2.9, 8.2, 6.5, 5.7, 3.0, 4.0, 0.1, 11.8, 14.2, 2.4, 1.6, 13.3, 6.5, 8.3, 4.9, 1.5, 18.6, 0.4, 0.4, 0.2, 11.6, 3.2, 7.1, 10.7, 3.9, 6.1, 6.4, 3.8, 15.2, 3.5, 3.1, 7.9, 18.2, 10.1, 4.4, 1.3, 13.7, 6.3, 3.6, 9.0, 7.7, 4.9, 9.1, 3.3, 8.5, 6.1, 0.4, 9.3, 0.5, 1.2, 1.7, 4.5, 3.1, 3.1, 6.6, 4.4, 5.0, 3.2, 7.7, 18.2, 4.1

The SNH distribution is validated to many other competing models, including the inverse Weibull (IW) by [29], Weibull (W) by [30], Kumaraswamy exponentiated Burr XII (KEBII) by [31], Weibull-Lomax (WL) by [32], Marshall-olkin alpha power inverse Weibull (MOAPIW) [33], Kumaraswamy Weibull (KW) by [34], and extended odd Weibull Lomax (EOWL) by [35].

Table gives the MLE estimates and standard errors (SE) for all model parameters, as well as the P-values (PVKS) for the Kolmogorov-Smirnov distance (KSD) statistics, the Akaike information criterion (AIC), the corrected AIC (CAIC), the Bayesian information criterion (BIC), the Hannan-Quinn information criterion (HQIC), the Cramer-von Mises (CVM), and the Anderson–Darling (AD) for all competitive models using the data used in the application section.

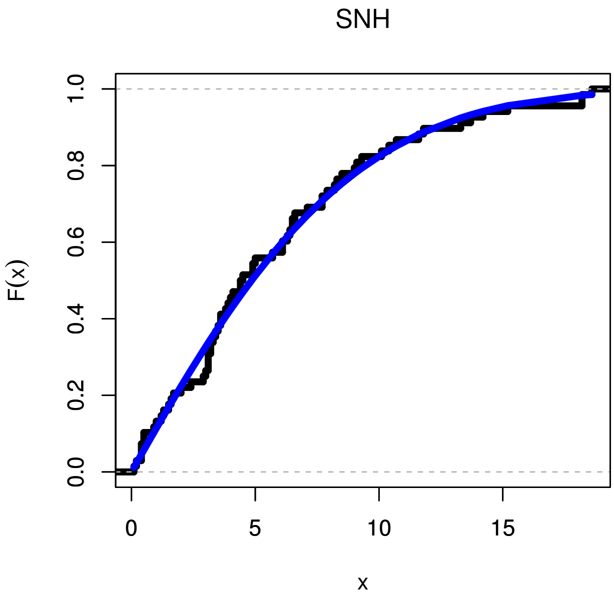

Figure 2. Estimated cdf for SNH distribution of data

The curve of the estimated cdf of SNH distribution is displayed over the ones of the corresponding empirical cdf of the data in Figure 2, together with the histograms and the curves of the associated

Table 4. MLE with different measures

		estimates	SE	AIC	CAIC	BIC	HQIC	CVM	AD	KSD	PVKS
SNH	α	3.9642	1.2667	375.2964	375.4810	379.7354	377.0553	0.0401	0.2564	0.0817	0.7544
	λ	0.0185	0.0219								
NH	α	3.0570	2.3885	375.3033	375.4879	379.7423	377.0621	0.0415	0.2641	0.0846	0.7149
	λ	0.0388	0.0366								
IW	θ	0.7428	0.0603	423.9449	424.1295	428.3839	425.7038	0.7535	4.2158	0.2218	0.0025
	β	1.7266	0.2094								
W	θ	1.2248	0.1189	376.3396	376.5242	380.7786	378.0985	0.0487	0.3191	0.0886	0.6593
	β	6.2367	0.6484								
KEBXII	a	59.6973	48.3504	386.1400	387.1078	397.2376	390.5372	0.1057	0.6749	0.1159	0.3205
	λ	2.6264	2.1250								
	b	878.8083	1515.0922								
	β	0.0734	0.0294								
	δ	4.1593	1.6381								
WL	α	0.6065	3.6002	379.3216	379.9565	388.1996	382.8394	0.0404	0.2567	0.0820	0.7509
	λ	1.0039	0.3082								
	θ	2.0828	4.4869								
	β	10.9628	81.0411								
MOAPIW	α	270.6825	480.3184	394.7630	395.3979	403.6410	398.2807	0.2497	1.5459	0.1343	0.1718
	λ	1.5551	0.1454								
	θ	107.6239	56.3932								
	β	0.0147	0.0075								
KW	α	0.8749	0.0035	407.9942	408.6292	416.8723	411.5120	0.1003	0.6444	0.2483	0.0005
	λ	0.0650	0.0079								
	θ	10.0240	0.0032								
	β	0.6845	0.0022								
EOWL	α	1.1741	0.5868	378.6758	379.3108	387.5539	382.1936	0.0963	0.5542	0.0617	0.3857
	λ	-0.4967	0.5591								
	θ	0.4642	0.8624								
	β	2.5411	7.9689								

estimated pdfs in Figure 3. Quarantines-quarantines (Q-Q) of SNH distribution is provided in Figure 4, and Probability-Probability (PP) plot of SNH distribution in Figure 5 is provided.

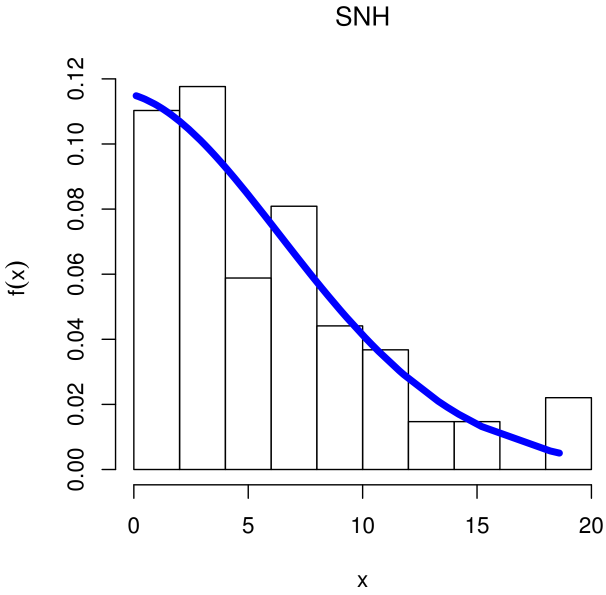

Figure 3. Estimated pdf for SNH distribution of data

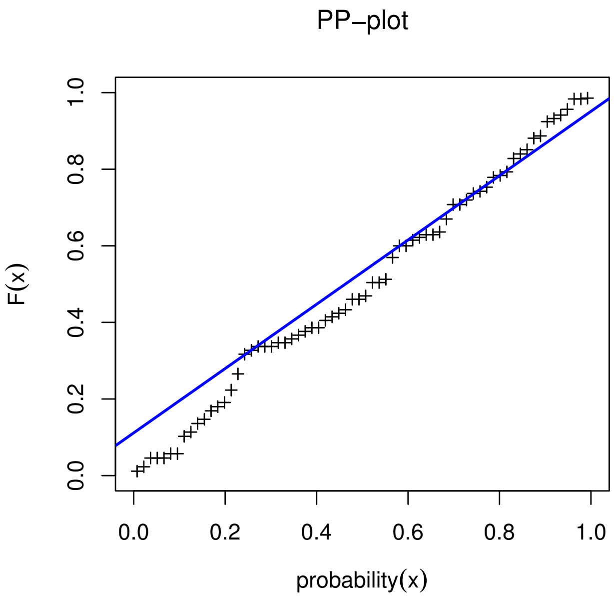

Figure 5. PP-plot for SNH distribution of data

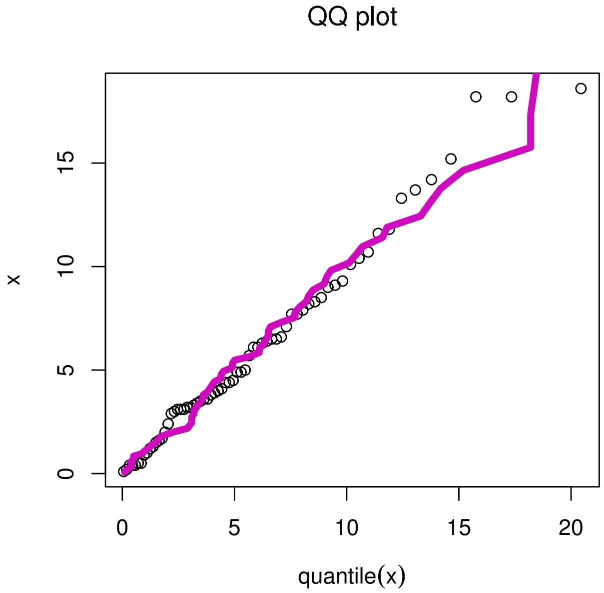

Figure 4. QQ-plot for SNH distribution of data

6. Conclusions

The Sine Nadarajah-Haghighi distribution is a brand-new distribution that is presented in this article. Some properties of the proposed distribution such as survival, hazard rate, reversed hazard rate, the first quintile, minimum, median, mean, third quintile, maximum, and standard deviation are some of the properties of the suggested distribution that are reviewed. The MLE method and MPS approach are used to estimate the model parameters. Also, we concluded the MPS is better than MLE for estimating parameters of SNH distribution. SNH model is used to fit a set of environmental data. To put it more succinctly, we anticipate that the proposed distribution and its participants will be attractive for widespread applications in a variety of industries, including insurance, bio-informatics, economics, queuing theory, meteorology, and hydrology.

Conflict of interest

The authors state that they have no financial or other conflicts of interest to disclose with connection to this research.

References

1. Kumar, D., Singh, U., & Singh, S. K. (2015). A new distribution using sine function-its application to bladder cancer patients data. Journal of Statistics Applications & Probability, 4(3), 417.
2. Al-Babtain, A. A., Elbatal, I., Chesneau, C., & Elgarhy, M. (2020). Sine Topp-Leone-G family of distributions: Theory and applications. Open Physics, 18(1), 574-593.

3. Souza, L., Junior, W., De Brito, C., Chesneau, C., Ferreira, T., & Soares, L. (2019). On the Sin-G class of distributions: theory, model and application. Journal of Mathematical Modeling, 7(3), 357-379.

4. Jamal, F., Chesneau, C., & Aidi, K. (2021). The sine extended odd Fréchet-G family of distribution with applications to complete and censored data. Mathematica Slovaca, 71(4), 961-982.

5. Mahmood, Z., Chesneau, C., & Tahir, M. H. (2019). A new sine-G family of distributions: properties and applications. Bull. Comput. Appl. Math., 7(1), 53-81.

6. Tomy, L., & Chesneau, C. (2021). The Sine Modified Lindley Distribution. Mathematical and Computational Applications, 26(4), 81.

7. Alyami, S. A., Elbatal, I., Alotaibi, N., Almetwally, E. M., & Elgarhy, M. (2022). Modeling to Factor Productivity of the United Kingdom Food Chain: Using a New Lifetime-Generated Family of Distributions. Sustainability, 14(14), 8942.

8. Fayomi, A., Algarni, A., & Almarashi, A. M. (2021). Sine Inverse Lomax Generated Family of Distributions with Applications. Mathematical Problems in Engineering, 2021.

9. Aldahlan, M. A. (2022). Sine Fréchet Model: Modeling of COVID-19 Death Cases in Kingdom of Saudi Arabia. Mathematical Problems in Engineering, 2022.

10. Ahmadini, A. A. H. (2022). Statistical Inference of Sine Inverse Rayleigh Distribution. Comput. Syst. Sci. Eng., 41(1), 405-414.

11. Nadarajah, S., & Haghighi, F. (2011). An extension of the exponential distribution. Statistics, 45(6), 543-558.

12. Gómez, Y. M., Bolfarine, H., & Gómez, H. W. (2014). A new extension of the exponential distribution. Revista Colombiana de Estadistica, 37(1), 25-34.

13. Khan, M. S., King, R., & Hudson, I. L. (2017). Transmuted generalized exponential distribution: A generalization of the exponential distribution with applications to survival data. Communications in Statistics-Simulation and Computation, 46(6), 4377-4398.

14. Muhammad, M. (2016). Poisson-odd generalized exponential family of distributions: Theory and applications. Hacettepe Journal of Mathematics and Statistics, 47(6), 1652-1670.

15. de Andrade, T. A., Bourguignon, M., & Cordeiro, G. M. (2016). The exponentiated generalized extended exponential distribution. Journal of Data Science, 14(3), 393-413.

16. Kumar, D., Dey, S., & Nadarajah, S. (2017). Extended exponential distribution based on order statistics. Communications in Statistics-Theory and Methods, 46(18), 9166-9184.

17. Hassan, A. S., Mohamd, R. E., Elgarhy, M., & Fayomi, A. (2018). Alpha power transformed extended exponential distribution: properties and applications. Journal of Nonlinear Sciences and Applications, 12(4), 62-67.

18. Peña-Ramírez, F. A., Guerra, R. R., Cordeiro, G. M., & Marinho, P. R. (2018). The exponentiated power generalized Weibull: Properties and applications. Anais da Academia Brasileira de Ciências, 90, 2553-2577.

19. Patil, G. P., Patil, G. P., & Rao, C. R. (1994). Handbook of statistics 12: Environmental statistics.

20. Cheng, R. C. H., & Amin, N. A. K. (1983). Estimating parameters in continuous univariate distributions with a shifted origin. Journal of the Royal Statistical Society: Series B (Methodological), 45(3), 394-403.
21. Ranneby, B. (1984). The maximum spacing method. An estimation method related to the maximum likelihood method. Scandinavian Journal of Statistics, 11, 93-112.
22. Coolen, F. P. A., & Newby, M. J. (1990). A note on the use of the product of spacings in Bayesian inference. Department of Mathematics and Computing Science, University of Technology.
23. Ghosh, K., & Jammalamadaka, S. R. (2001). A general estimation method using spacings. Journal of Statistical Planning and Inference, 93(1-2), 71-82.
24. Rahman, M., & Pearson, L. M. (2001). Estimation in two-parameter exponential distributions. Journal of Statistical Computation and Simulation, 70(4), 371-386.
25. Basu, S., Singh, S. K., & Singh, U. (2018). Bayesian inference using product of spacings function for Progressive hybrid Type-I censoring scheme. Statistics, 52(2), 345-363.
26. Alshenawy, R., Al-Alwan, A., Almetwally, E. M., Afify, A. Z., & Almongy, H. M. (2020). Progressive type-II censoring schemes of extended odd Weibull exponential distribution with applications in medicine and engineering. Mathematics, 8(10), 1679.
27. Almetwally, E. M., Almongy, H. M., Rastogi, M. K., & Ibrahim, M. (2020). Maximum product spacing estimation of Weibull distribution under adaptive type-II progressive censoring schemes. Annals of Data Science, 7(2), 257-279.
28. El-Sherpieny, E. S. A., Almetwally, E. M., & Muhammed, H. Z. (2020). Progressive Type-II hybrid censored schemes based on maximum product spacing with application to Power Lomax distribution. Physica A: Statistical Mechanics and its Applications, 553, 124251.
29. Khan, M. S., Pasha, G. R., & Pasha, A. H. (2008). Theoretical analysis of inverse Weibull distribution. WSEAS Transactions on Mathematics, 7(2), 30-38.
30. Hallinan Jr, A. J. (1993). A review of the Weibull distribution. Journal of Quality Technology, 25(2), 85-93.
31. Afify, A. Z., & Mead, M. E. (2017). On five-parameter Burr XII distribution: properties and applications. South African Statistical Journal, 51(1), 67-80.
32. Tahir, M. H., Cordeiro, G. M., Mansoor, M., & ZUBAİR, M. (2015). The Weibull-Lomax distribution: properties and applications. Hacettepe Journal of Mathematics and Statistics, 44(2), 455-474.
33. Basheer, A. M., Almetwally, E. M., & Okasha, H. M. (2021). Marshall-olkin alpha power inverse Weibull distribution: non bayesian and bayesian estimations. Journal of Statistics Applications & Probability, 10(2), 327-345.
34. Cordeiro, G. M., Ortega, E. M., & Nadarajah, S. (2010). The Kumaraswamy Weibull distribution with application to failure data. Journal of the Franklin Institute, 347(8), 1399-1429.
35. Alsuhabi, H., Alkhairy, I., Almetwally, E. M., Almongy, H. M., Gemeay, A. M., Hafez, E. H., ... & Sabry, M. (2022). A superior extension for the Lomax distribution with application to Covid-19 infections real data. Alexandria Engineering Journal, 61(12), 11077-11090.

Supplementary (if necessary)